REINFORCEMENT LEARNING

Gergely Neu
Univ. Pompeu Fabra

A PRIMAL-DUAL VIEW OF

REINFORCEMENT LEARNING

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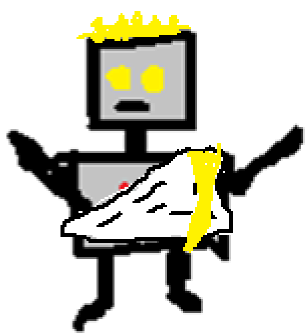
WHAT IS REINFORCEMENT LEARNING?

Agent



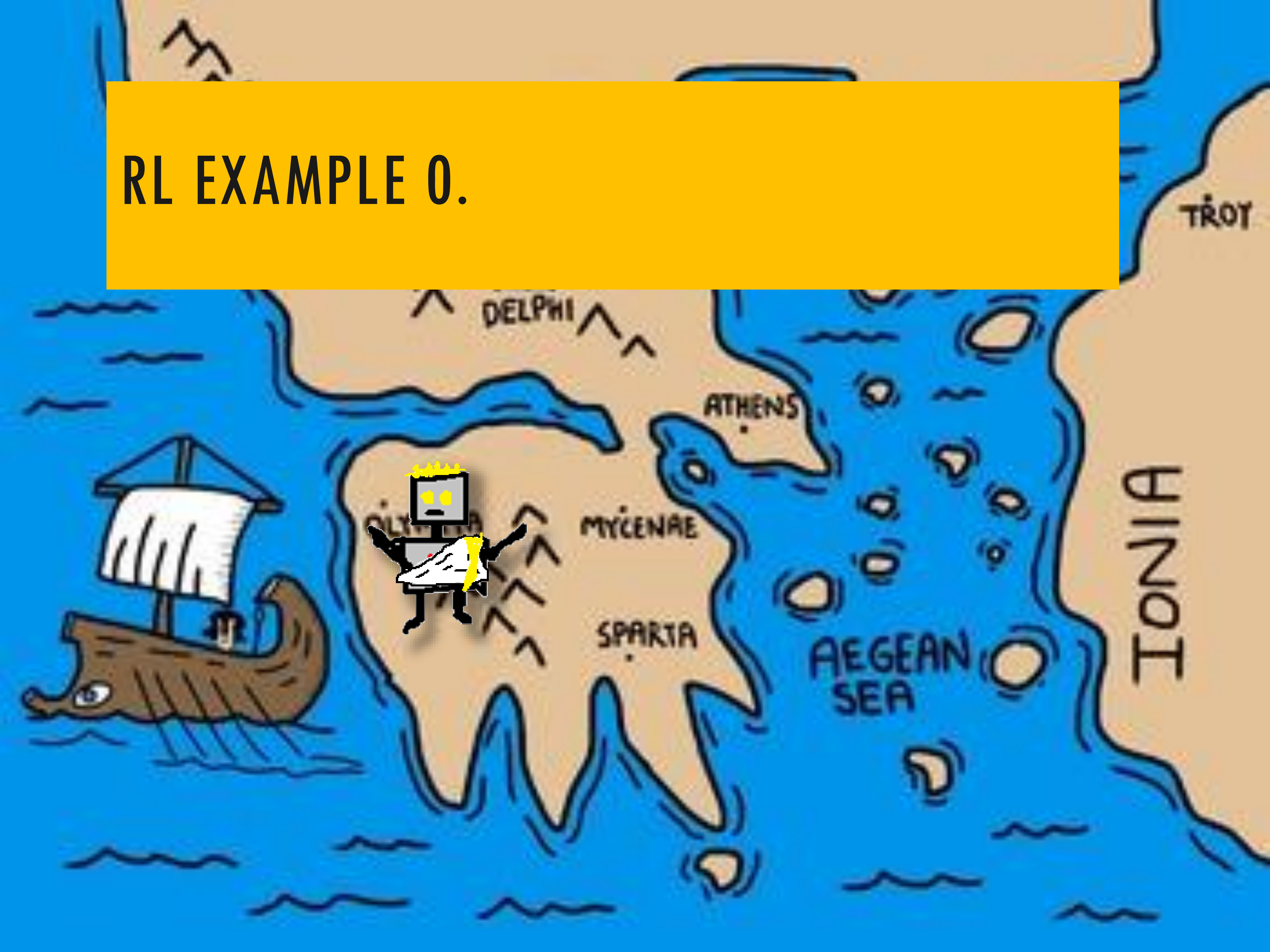
In state s,
take action a

Environment

Reward r,
new state s'

Learning to
- maximize reward
- in a reactive environment
- under partial feedback

RL EXAMPLE 0.



RL EXAMPLE 0.

RL EXAMPLE 0.


TROY
DELPHI
ATHENS
MYCENAE
SPARTA
AEGEAN SEA
IONIA
x
state
actions

RL EXAMPLE 0.


DELPHI
ATHENS
MYCENAE
SPARTA
AEGEAN SEA
IONIA
TROY
state
actions

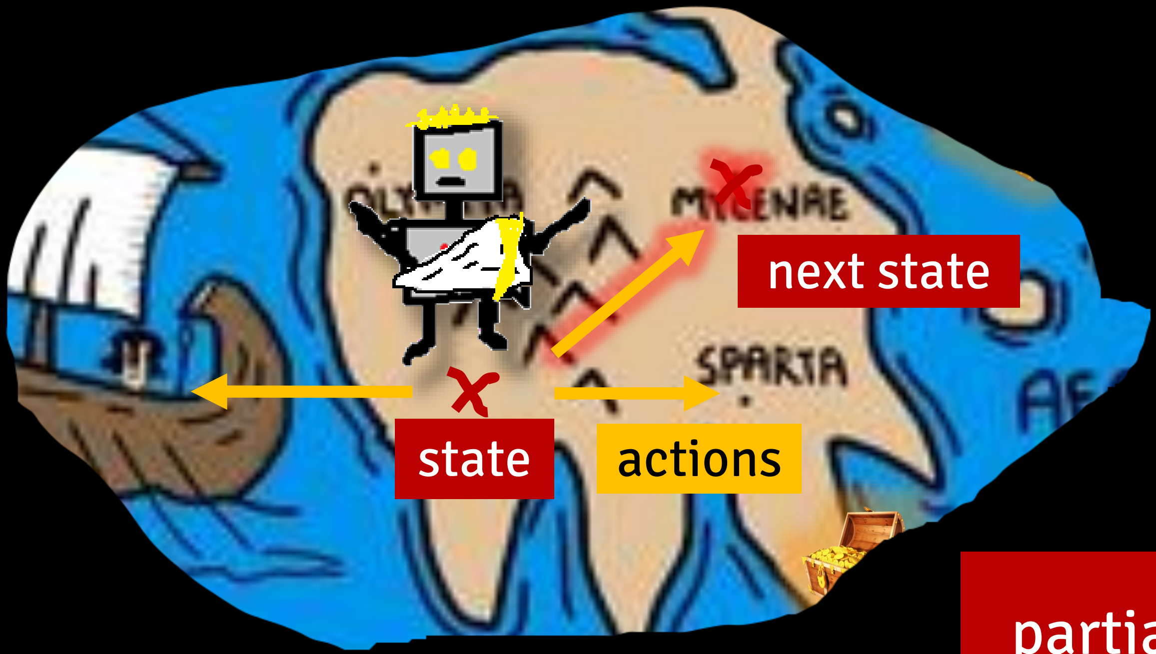
RL EXAMPLE 0.


DELPHI
ATHENS
TROY
next state
SPARTA
AEGEAN SEA
IONIA
state
actions

RL EXAMPLE 0.

DELPHI
ATHENS
TROY
reward
MYCENAE
next state
SPARTA
AEGEAN SEA
IONIA
state
actions

RL EXAMPLE 0.



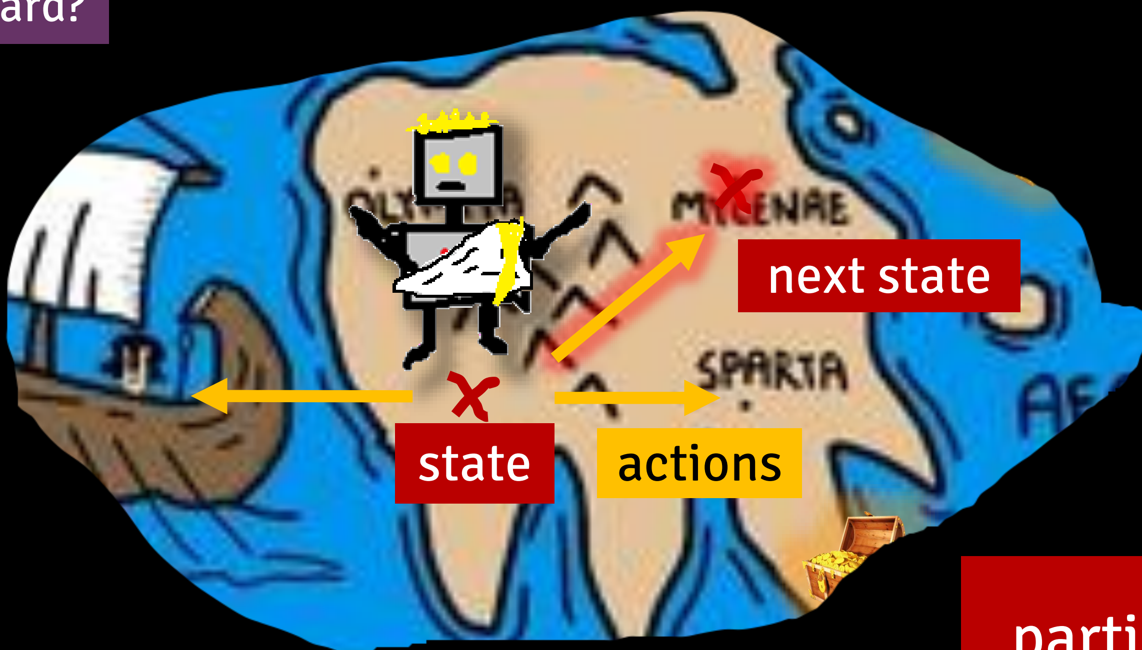
partial observability

RL EXAMPLE 0.

reward?

reward?

reward?



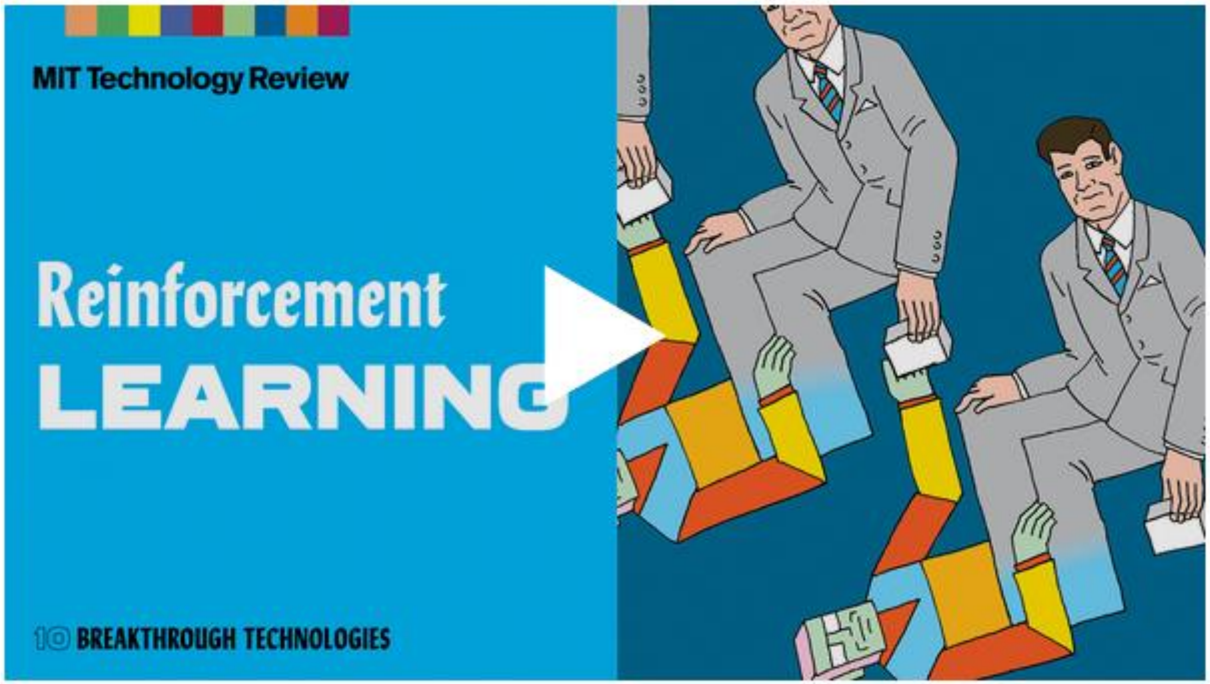
reward?

reward?

reward?

partial observability

WHY SHOULD I CARE?



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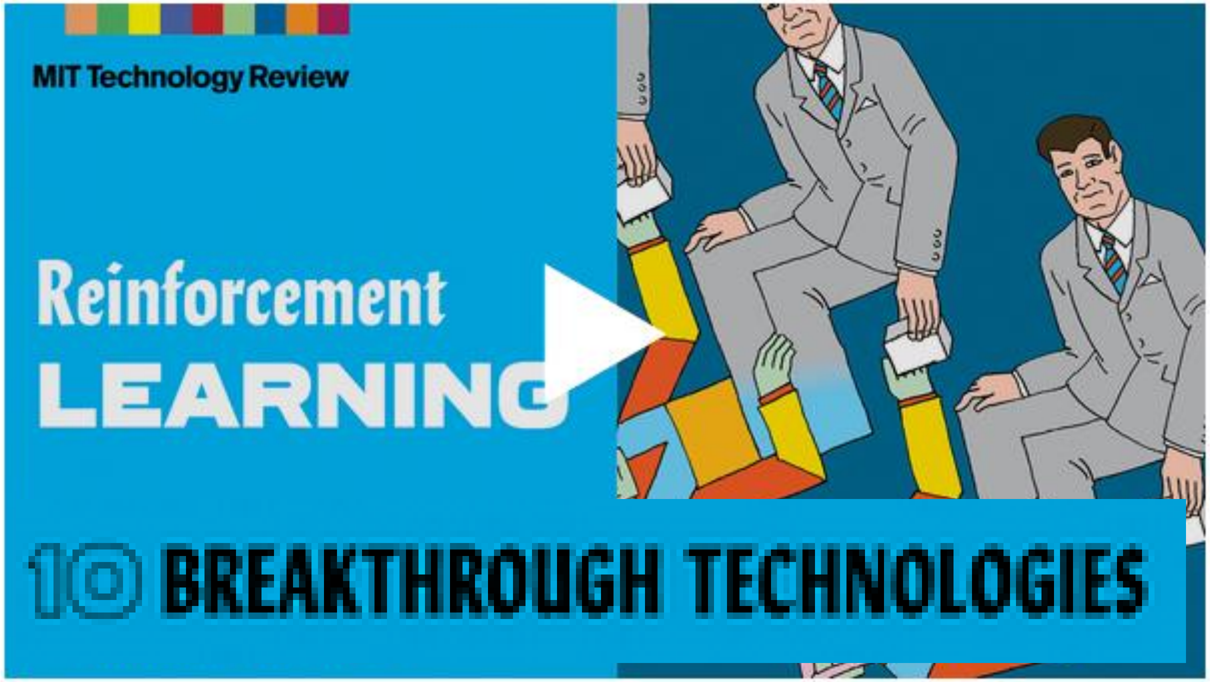

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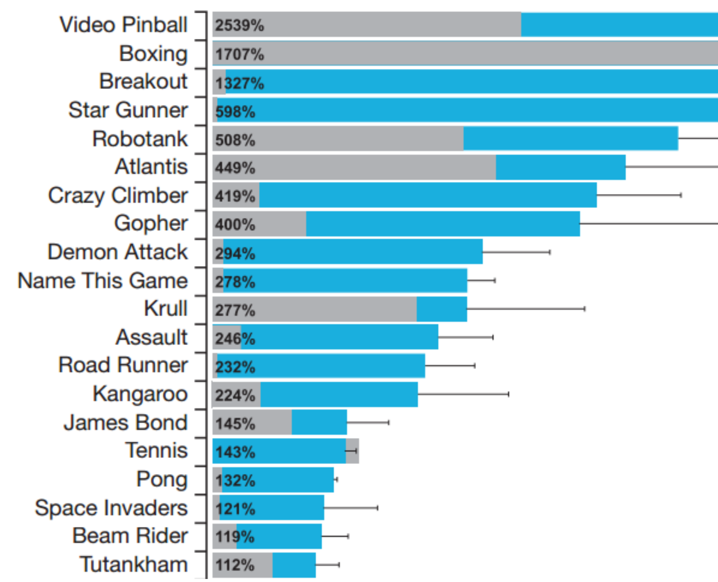
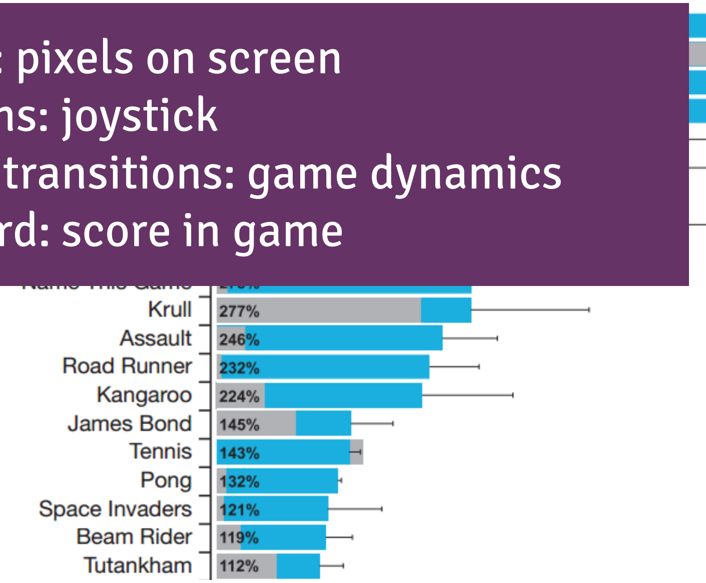


WHY SHOULD I CARE?

Breakthrough in Atari game playing

- State: pixels on screen
- Actions: joystick
- State transitions: game dynamics
- Reward: score in game



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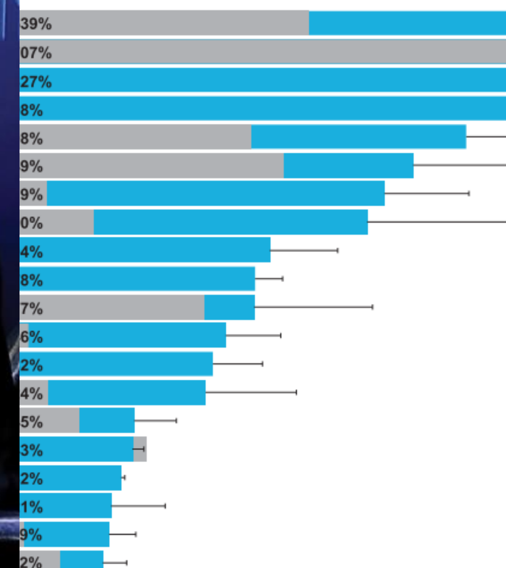
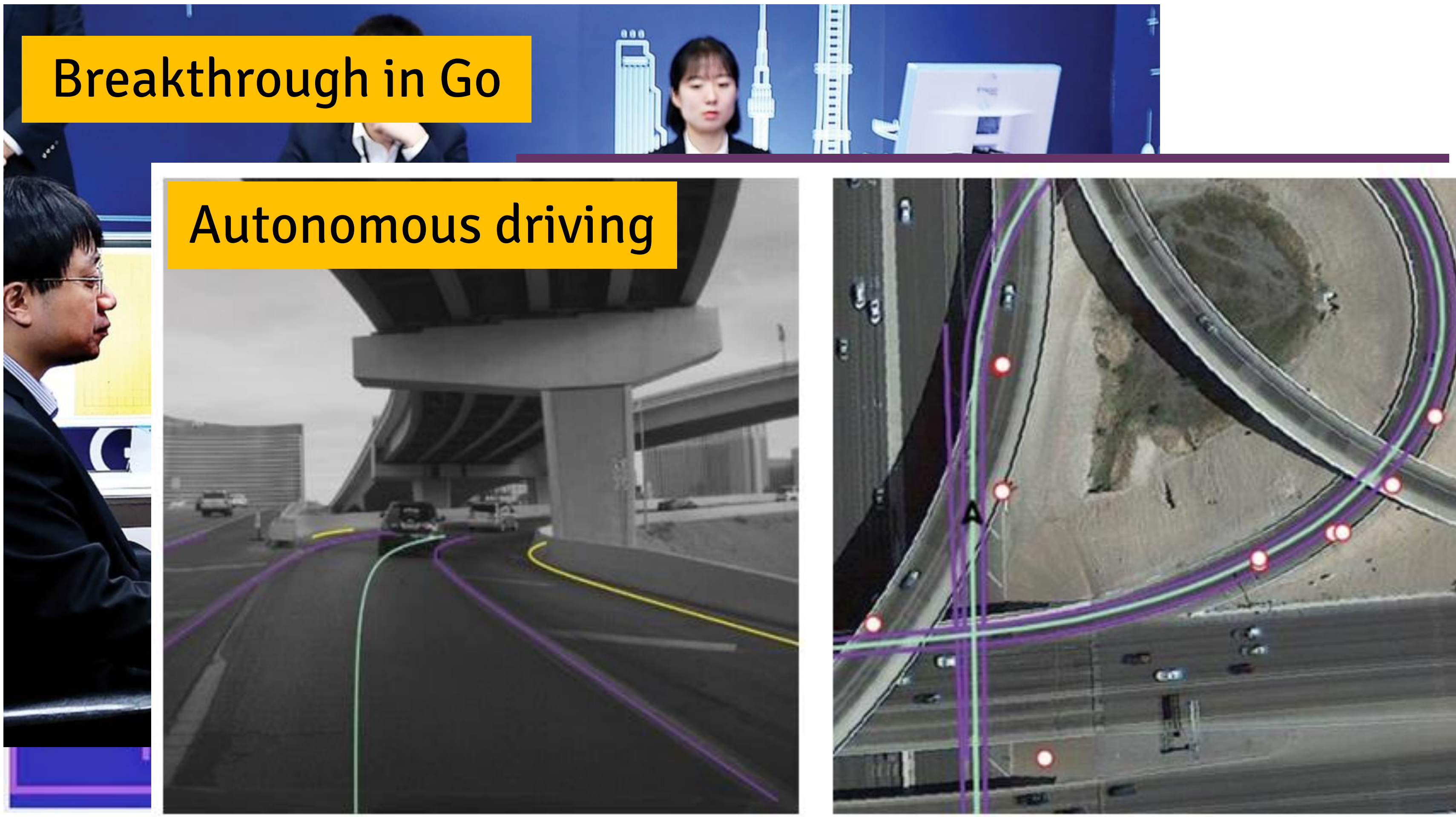


WHY SHOULD I CARE?

Breakthrough in Go

- State: stones currently on board
- Actions: place stone on board
- State transitions: own move + adversary's move
- Reward: +1 for winning the game

WHY SHOULD I CARE?

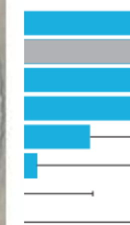


WHY SHOULD I CARE?

Breakthrough in Go

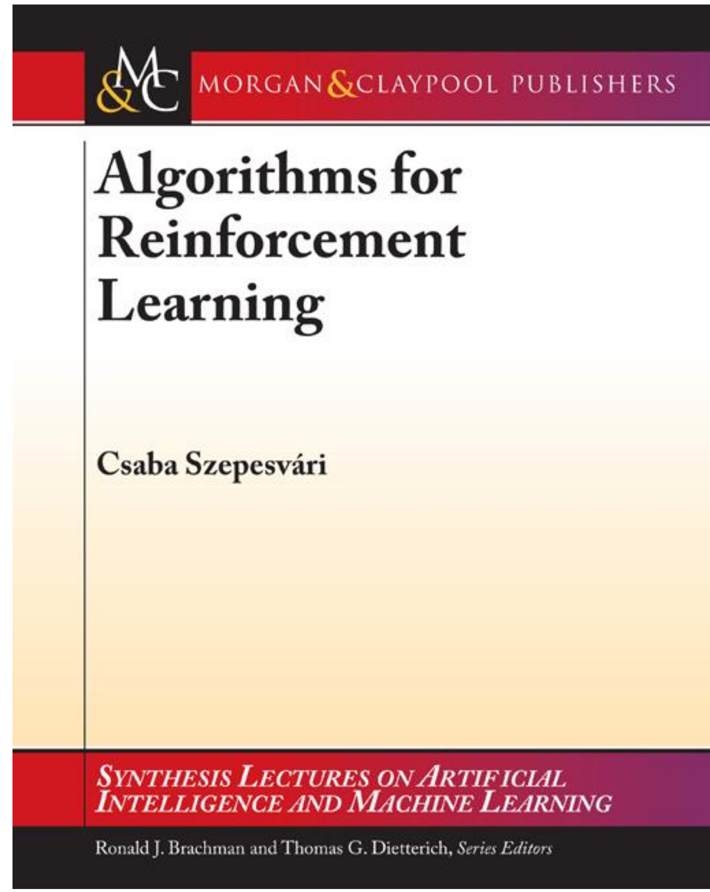
Autonomous driving

- State: road conditions, other vehicles, obstacles,…
- Actions: turn left/right, accelerate/brake,…
- State transitions: depending on state+action+randomness
- Reward: +100 for reaching destination, -100 for accidents,…



RECOMMENDED READING

- Richard Sutton and Andrew Barto (2018): “Reinforcement Learning: An Introduction”
  - For an enjoyable (but not very rigorous) introduction
- Dimitri Bertsekas (2012): “Dynamic Programming and Optimal Control”
  - For a rigorous treatment of the basics
- Csaba Szepesvári (2012): “Algorithms for RL”
  - For a rigorous description of basic RL algorithms



THIS SHORT COURSE: A PRIMAL-DUAL VIEW

- Markov decision processes
  - Value functions and optimal policies
- Primal view: Dynamic programming
  - Policy evaluation, value and policy iteration
  - Value-function-based methods
    - Temporal differences, Q-learning, LSTD, deep Q networks,…

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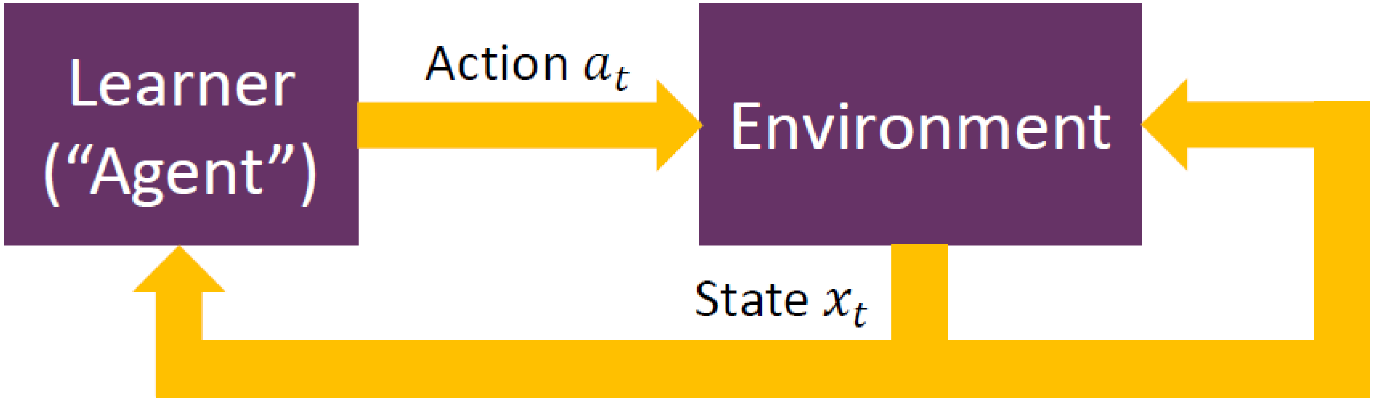
MARKOV DECISION PROCESSES (MDPs)



A Markov Decision Process (MDP) is characterized by

- X: a set of states
- A: a set of actions, possibly different in each state
- P: $X \times A \times X \to [0,1]$: a transition function with $P(\cdot\,|x,a)$ being the distribution of the next state given previous state x and action a:

$$\mathbf{P}[x_{t+1} = x' | x_t = x, a_t = a] = P(x'|x,a)$$

- r: $X \times A \to [0,1]$: a reward function

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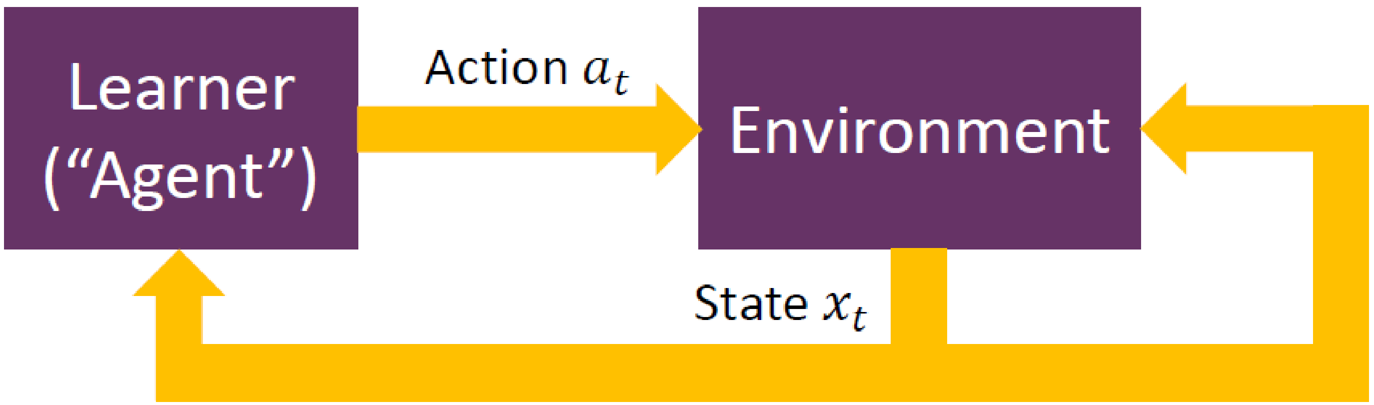

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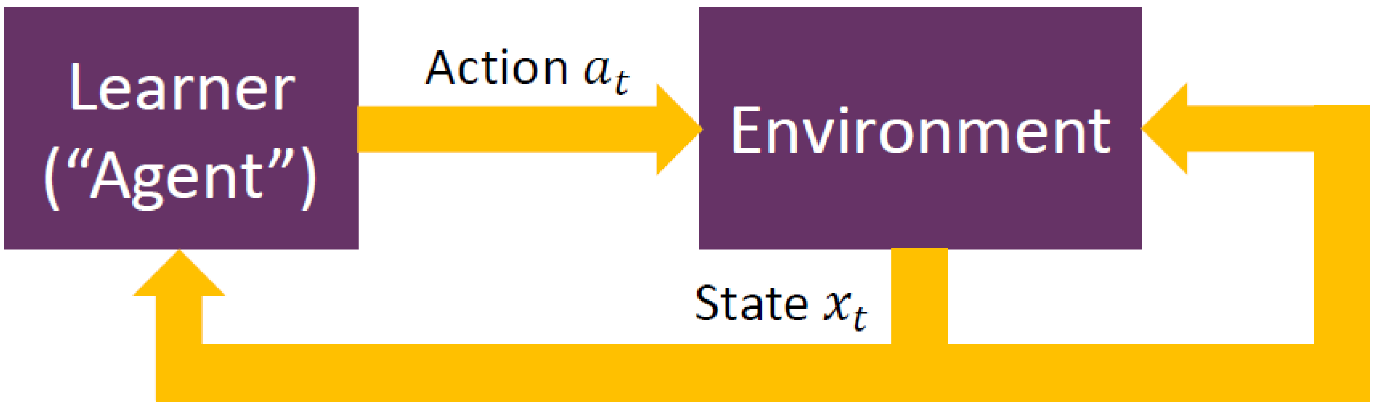

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Interaction in an MDP: in each round $t = 1,2, \ldots$

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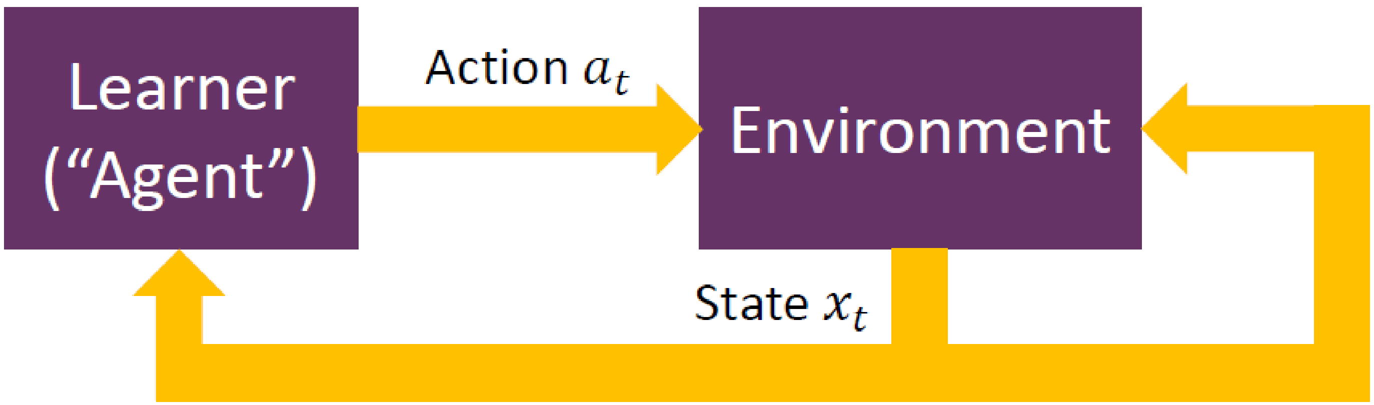

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GOAL:
maximize "total rewards"!

NOTIONS OF "TOTAL REWARD"

Episodic MDPs:

- There is a terminal state x^*
- GOAL: maximize total reward until final round T when x^* is reached:

$$R^* = \mathbf{E}[\sum_{t=0}^{T} r_t]$$

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Discounted MDPs:

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+ other notions:
- long-term average reward
- total reward up to fixed horizon
- …

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+ we will assume that X and A are finite

POLICIES AND TRAJECTORY DISTRIBUTIONS

Policy: mapping from histories to actions

$$\pi: x_1, a_1, x_2, a_2, \ldots, x_t \mapsto a_t$$

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Expectation under this distribution: $\mathbf{E}_\pi[\cdot]$

DEFINING OPTIMALITY

Optimal policy π^*: a policy that maximizes

$$\mathbf{E}_\pi\left[R_\gamma\right] = \mathbf{E}_\pi\left[\sum_{t=0}^{\infty} \gamma^t r_t\right]$$

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=“Markov property”

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Value function: evaluates policy π starting from state x:

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"Optimal policy π^* $= \arg\max_{\pi} V^{\pi}(x_0)$"

VALUE FUNCTIONS AND THE OPTIMAL POLICY

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The optimal value function:

$$V^* = V^{\pi^*}$$

THE BELLMAN EQUATIONS

Theorem

The value function of a stationary policy π satisfies the system of equations $(\forall x \in X)$

$$V^{\pi}(x) = r\big(x, \pi(x)\big) + \gamma \sum_y P(y|x, \pi(x))\, V^{\pi}(y)$$

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Theorem

The optimal value function satisfies the system of equations

$$V^*(x) = \max_a \left\{ r(x, a) + \gamma \sum_y P(y|x, a)\, V^*(y) \right\}$$

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Theorem

An optimal policy π^* satisfies

$$\pi^*(x) \in \arg\max_a \left\{ r(x,a) + \gamma \sum_y P(y|x,a)\, V^*(y) \right\}$$

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= greedy with respect to Q^*

SHORT SUMMARY SO FAR

So far, we have characterized

- The value functions of a given policy
- The optimal policy through value functions
- The optimal value functions
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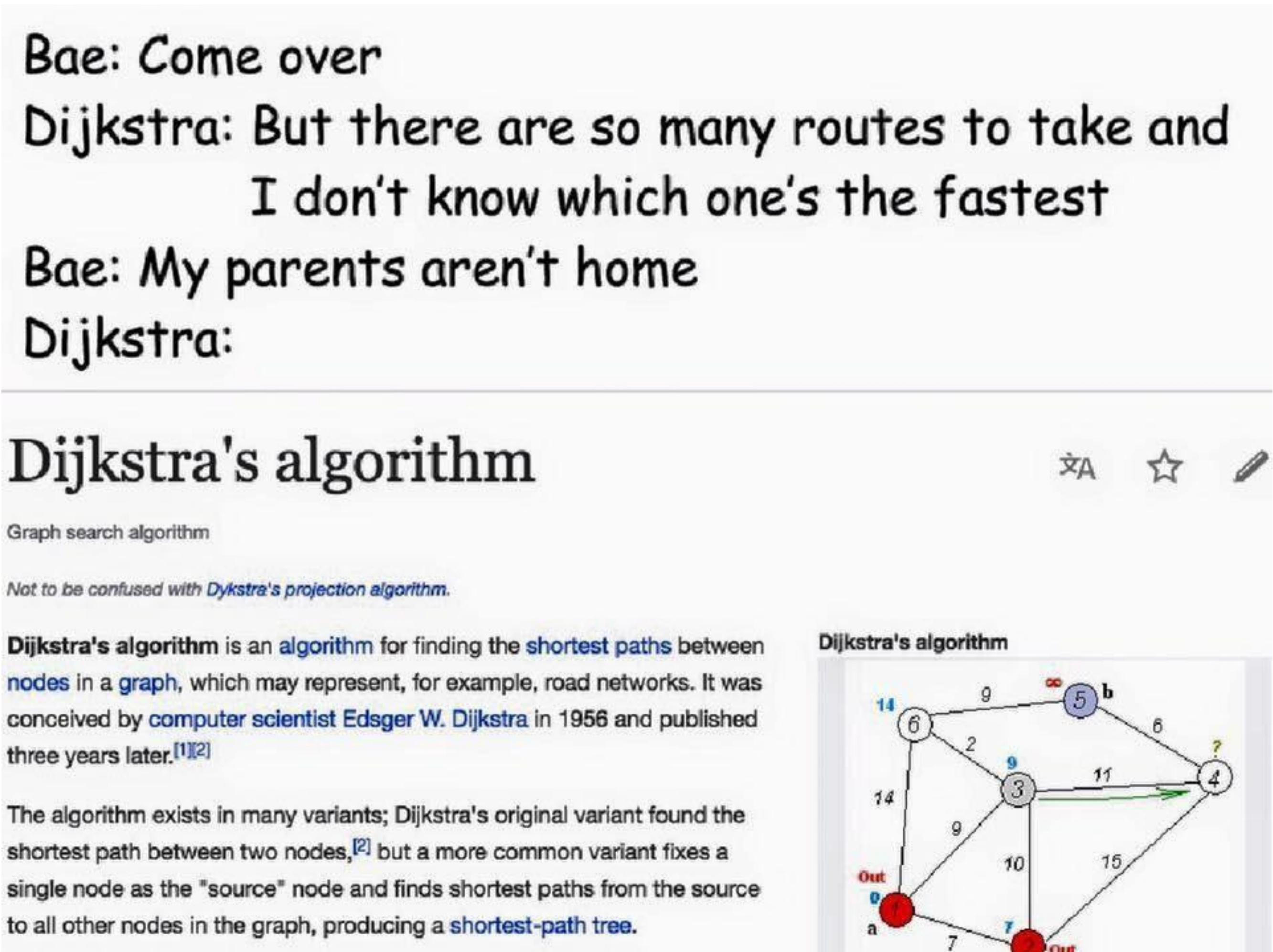
BUT HOW DO WE FIND THE OPTIMAL VALUE FUNCTION??

… also, is there a way to clean up this mess? See part 2!

EASY ANSWER FOR FINITE-HORIZON PROBLEMS



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DYNAMIC PROGRAMMING

Dynamic programming

=

computing value functions through repeated use of the "Bellman operators"

THE BELLMAN OPERATOR

Bellman operator T^{π}:
maps a function $V \in \mathbb{R}^X$ to another function $T^{\pi}V \in \mathbb{R}^X$:

$$(T^{\pi}V)(x) = r\big(x, \pi(x)\big) + \gamma \sum_y P(y|x, \pi(x))V(y)$$

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r.h.s. of BE

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The Bellman Equations:

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CONVERGENCE OF POWER ITERATION: PROOF SKETCH

- Power iteration can be written as the linear recursion
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Geometric sum!
(von Neumann series)

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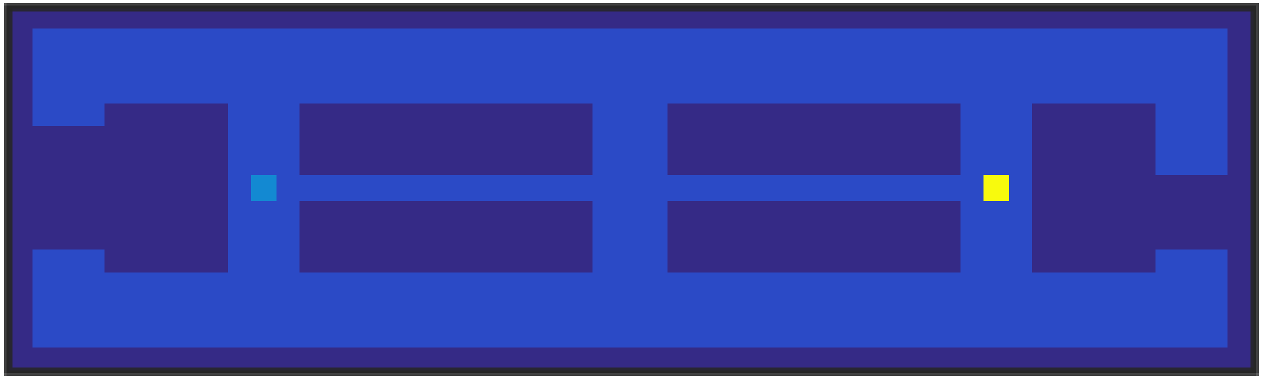
- The value function V^π satisfies

$$V^\pi = r + \gamma P^\pi V^\pi \iff V^\pi = (I - \gamma P^\pi)^{-1} r$$

POWER ITERATION IN ACTION

Gridworld MDP



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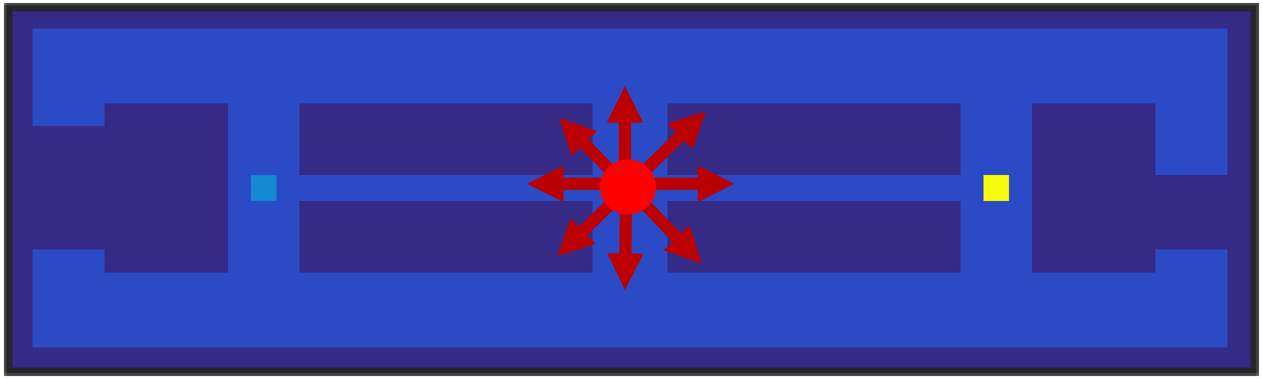

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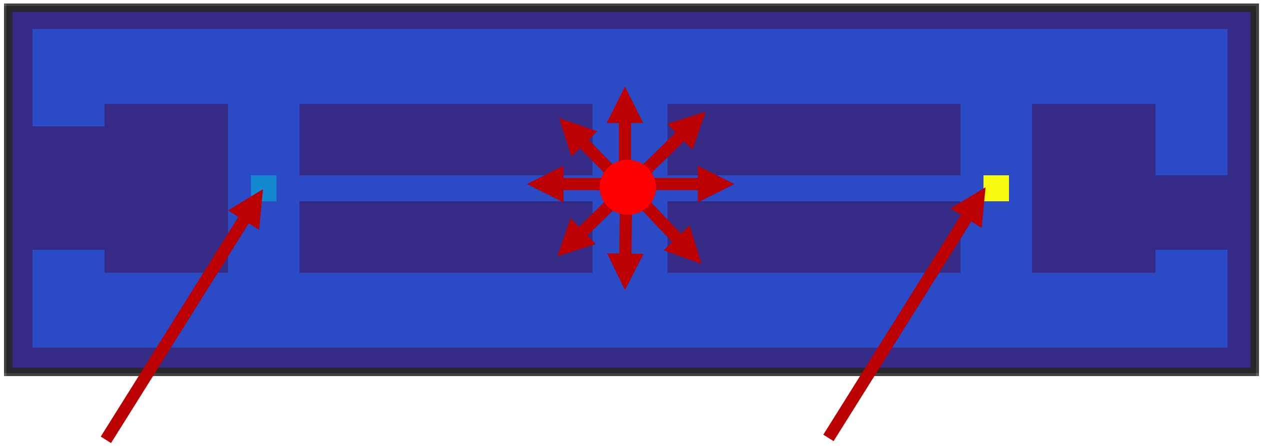

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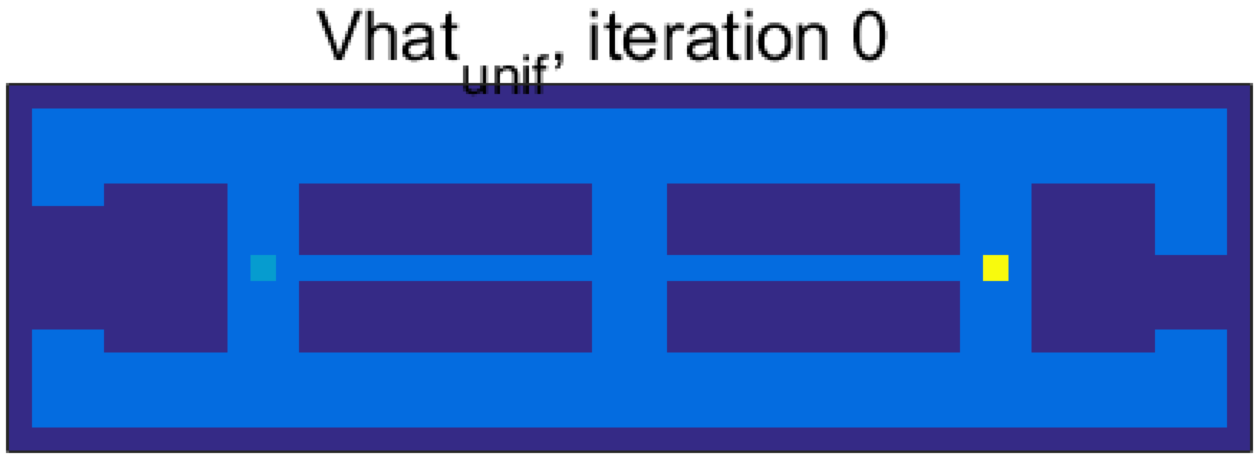

Uniform policy:

$$\pi(a|x) = \frac{1}{9}$$

for all actions $a \in \{1,2,\dots,9\}$

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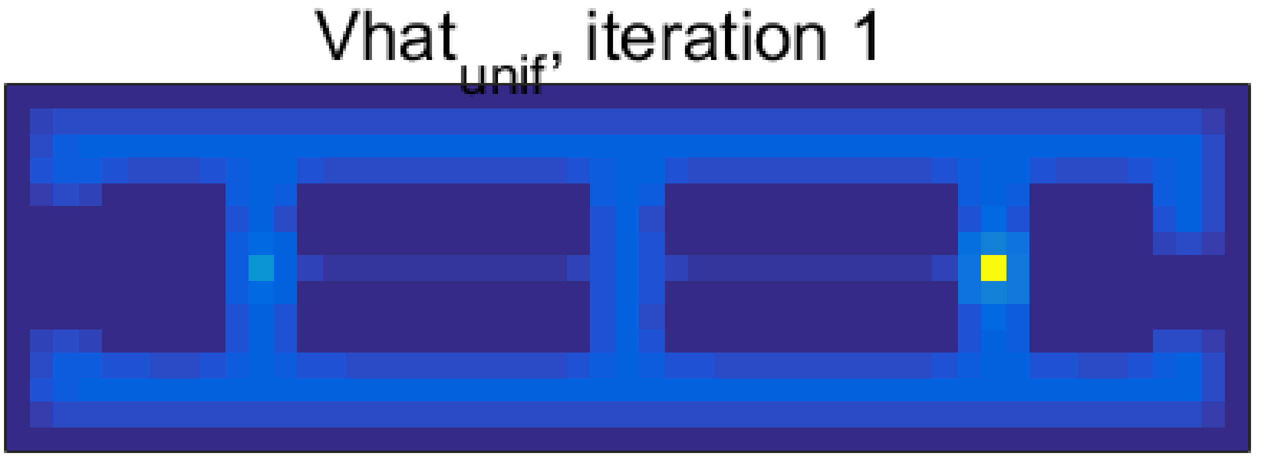

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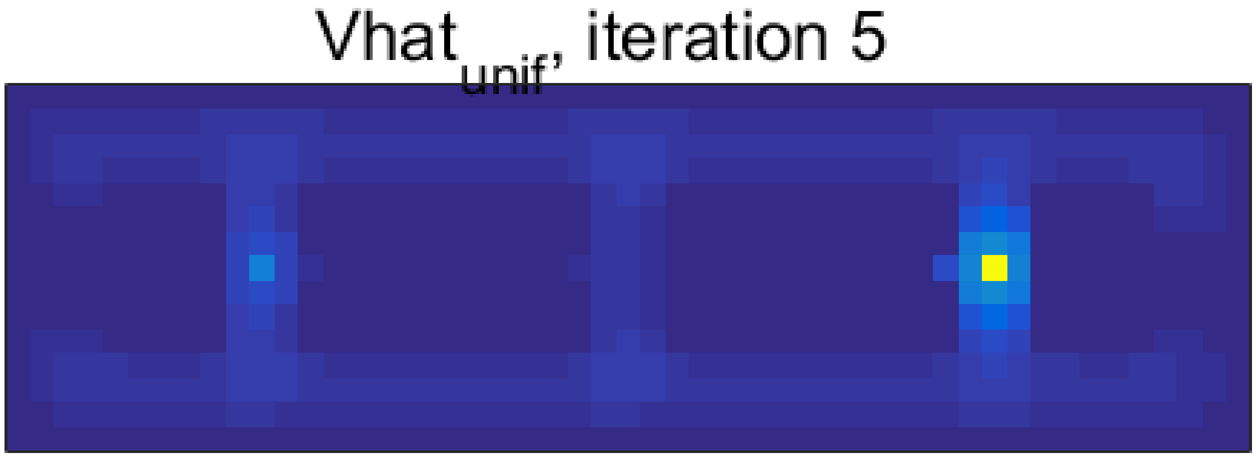

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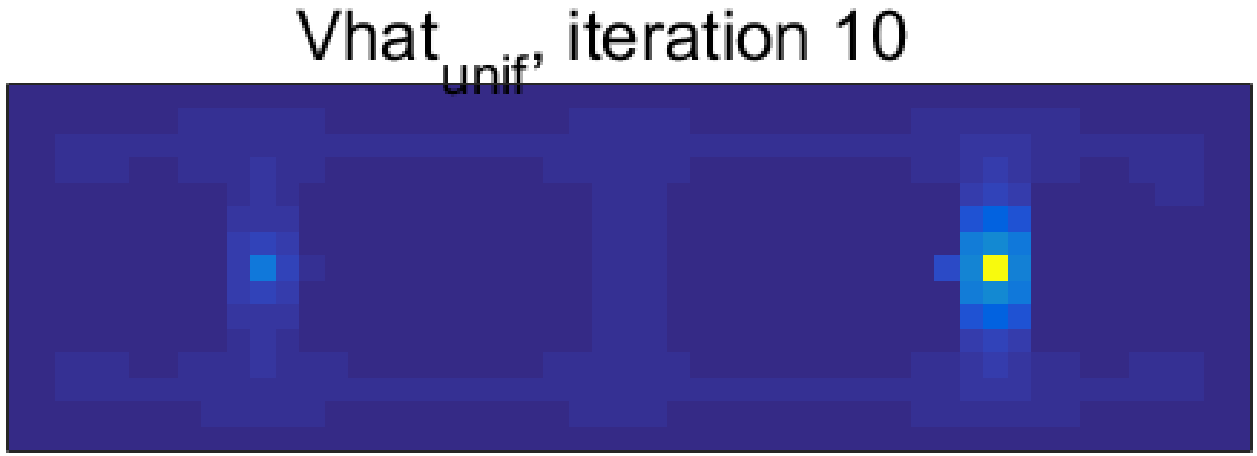

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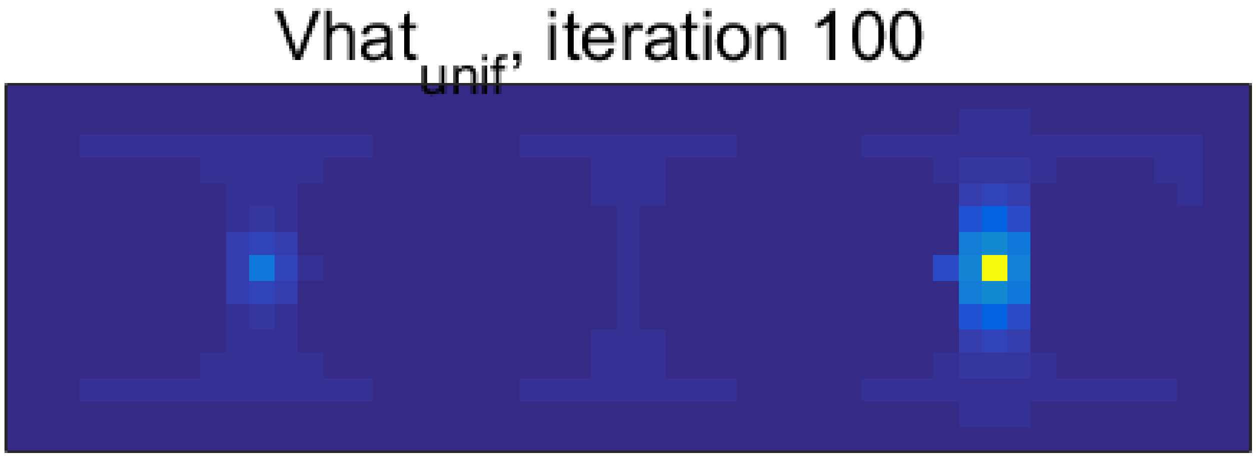

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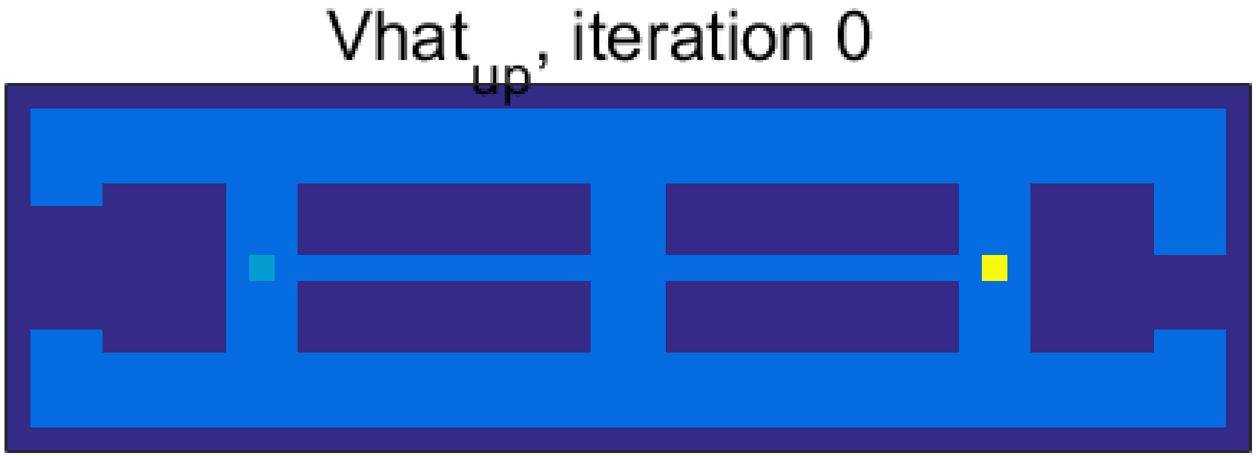

“Upwards” policy:

$\pi(\mathrm{up}|x) = 1$

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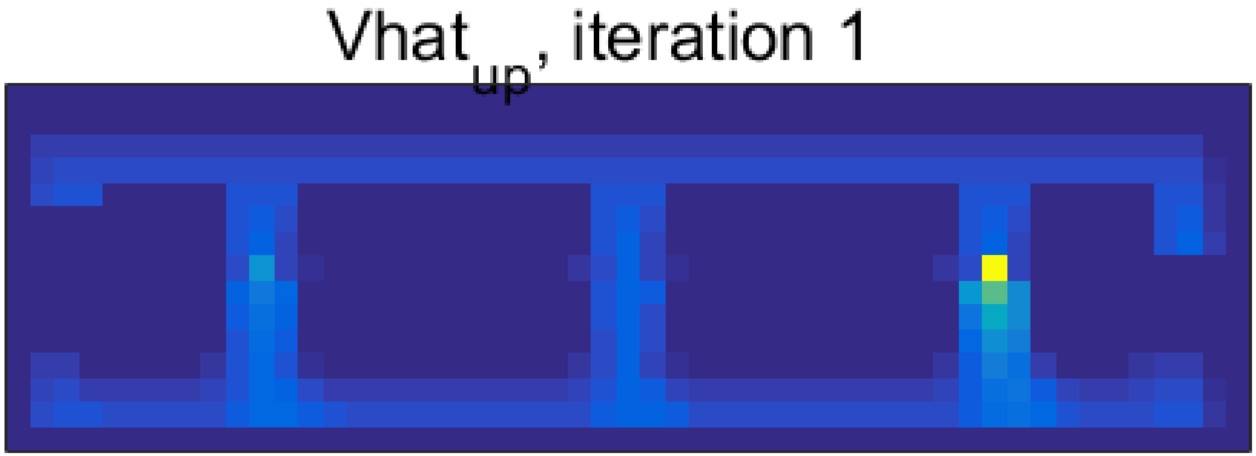

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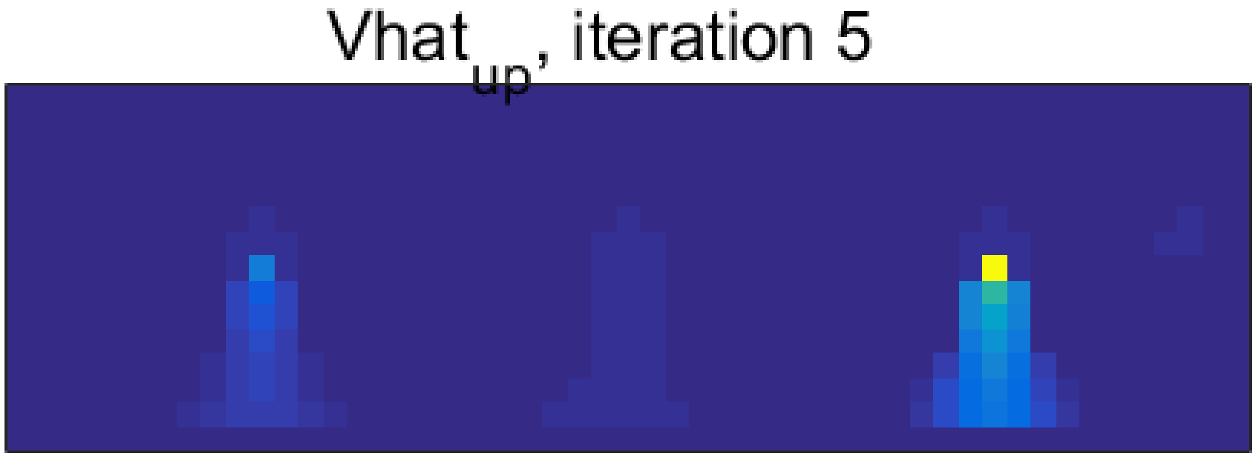

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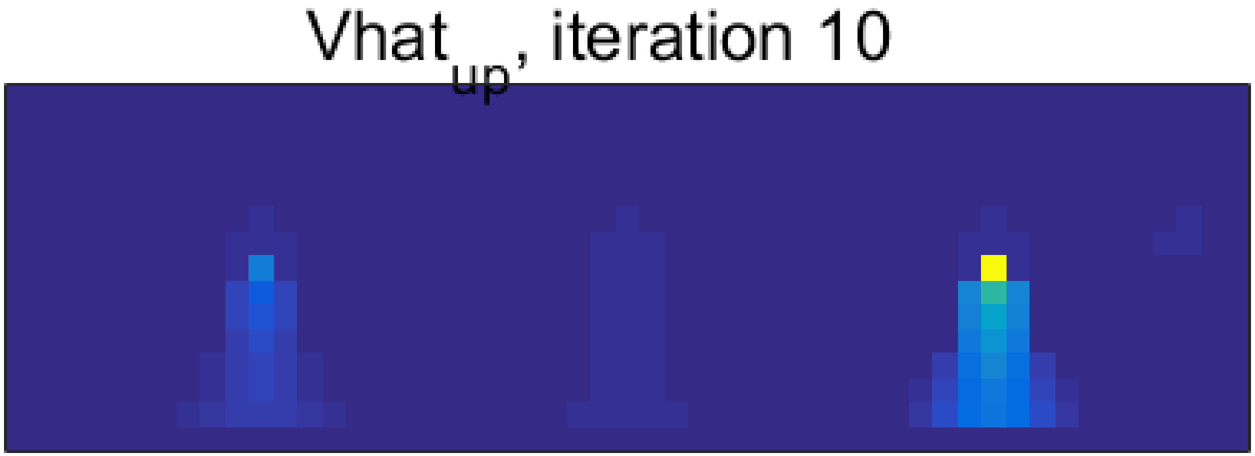

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THE BELLMAN OPTIMALITY OPERATOR

Bellman optimality operator T^*:
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Key idea: T^* is a contraction

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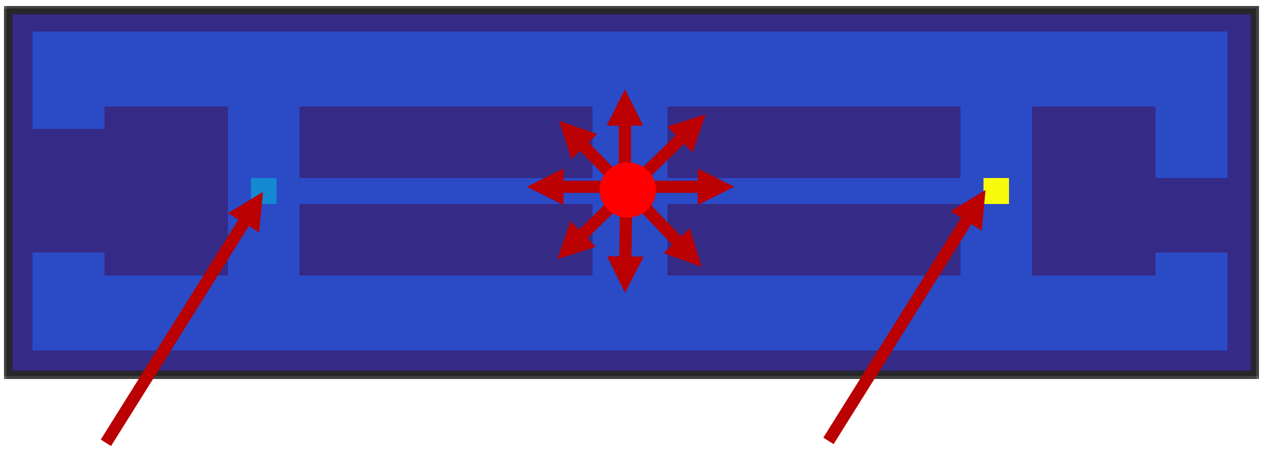

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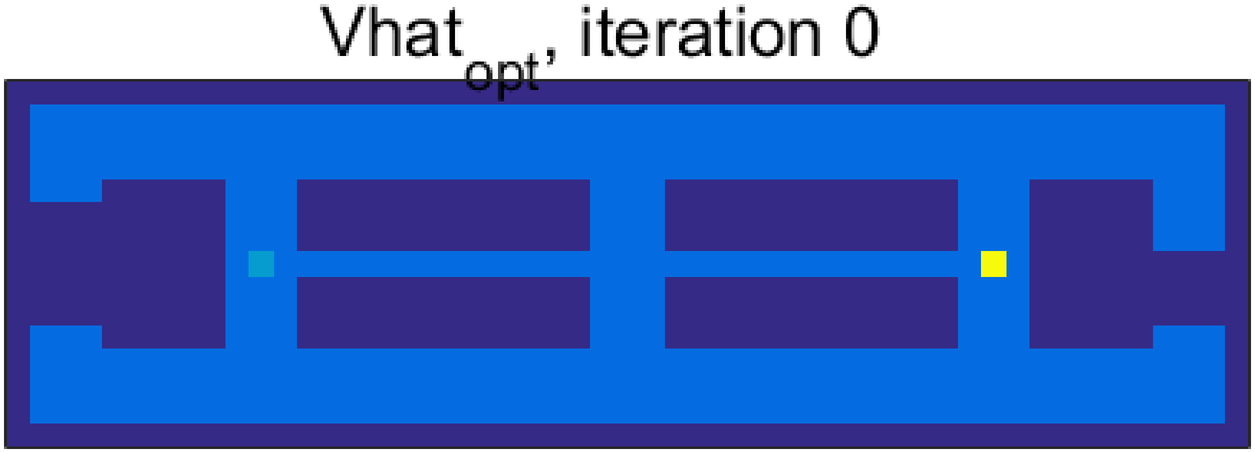

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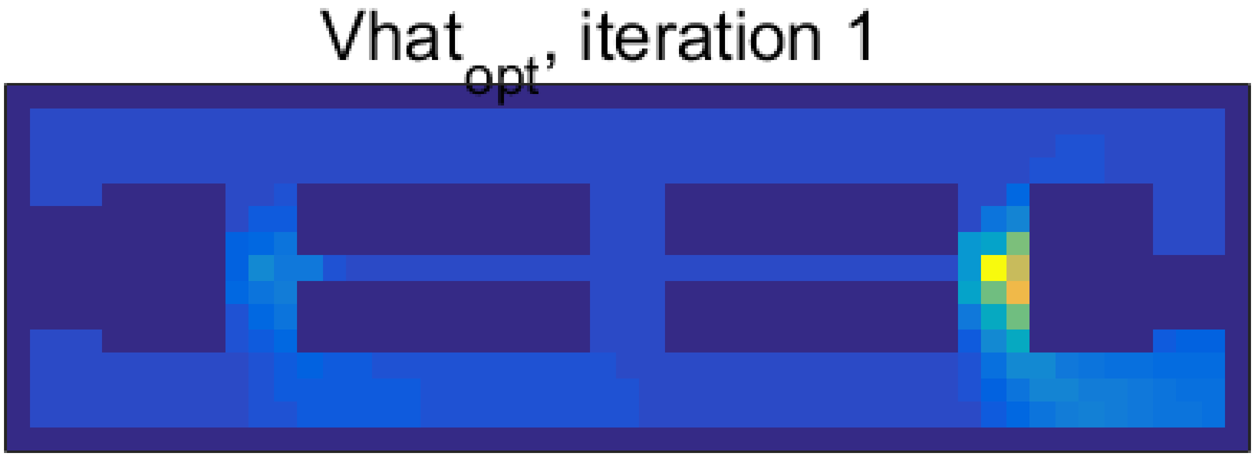

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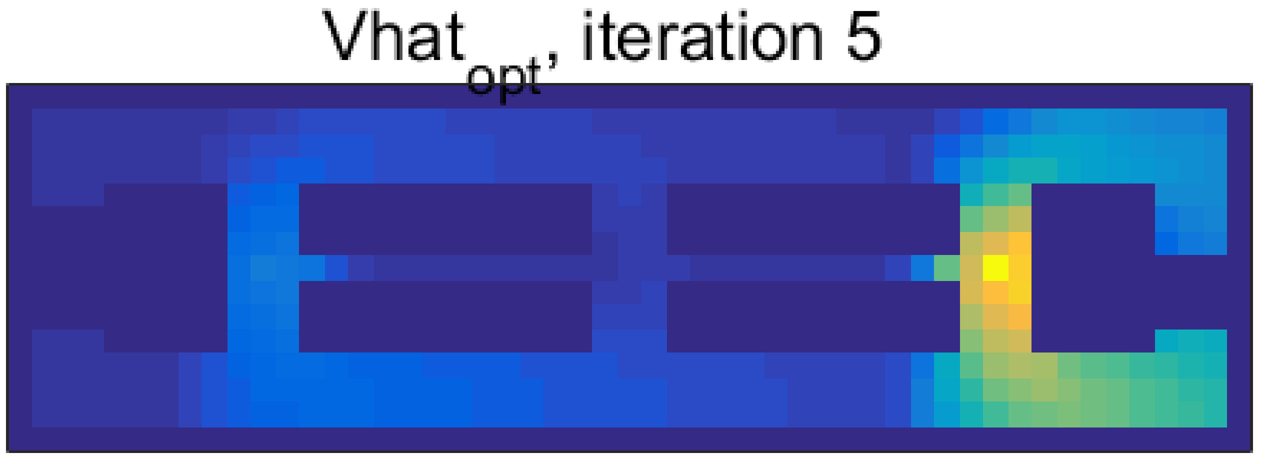

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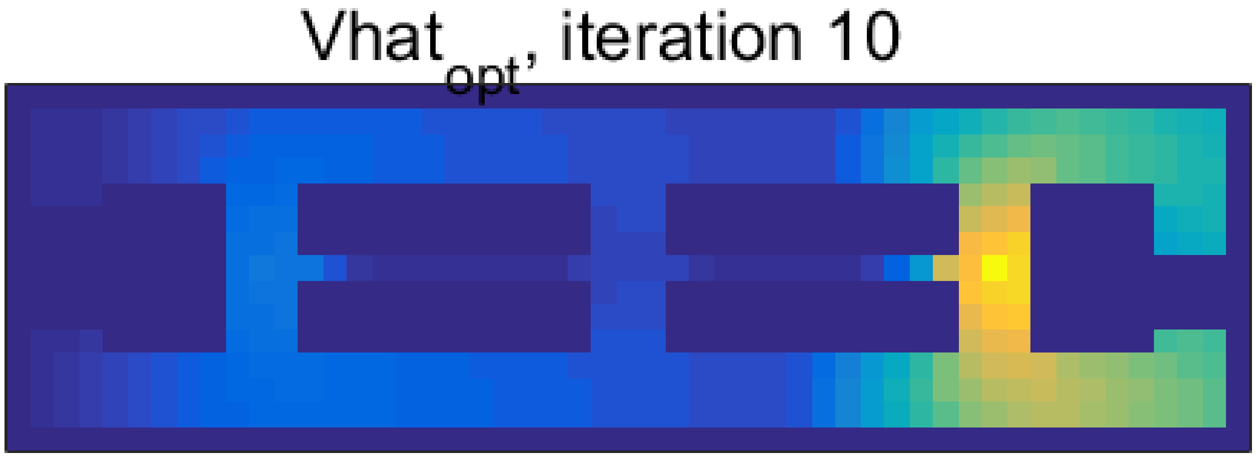

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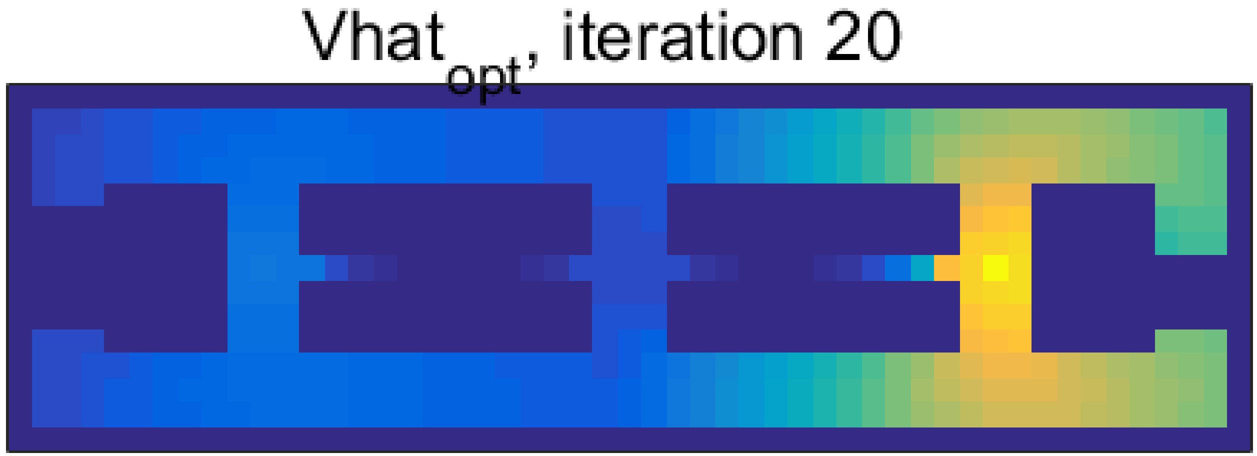

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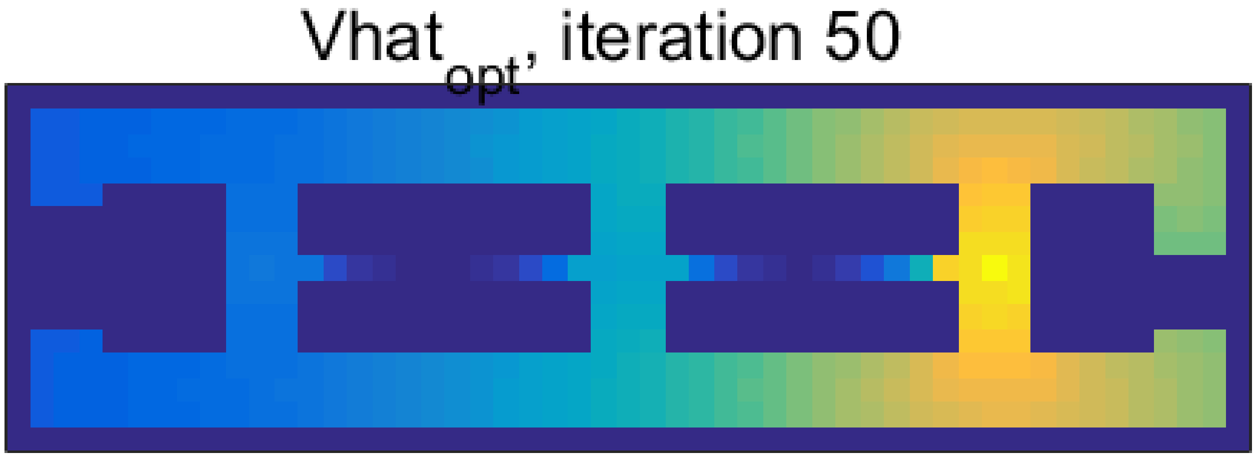

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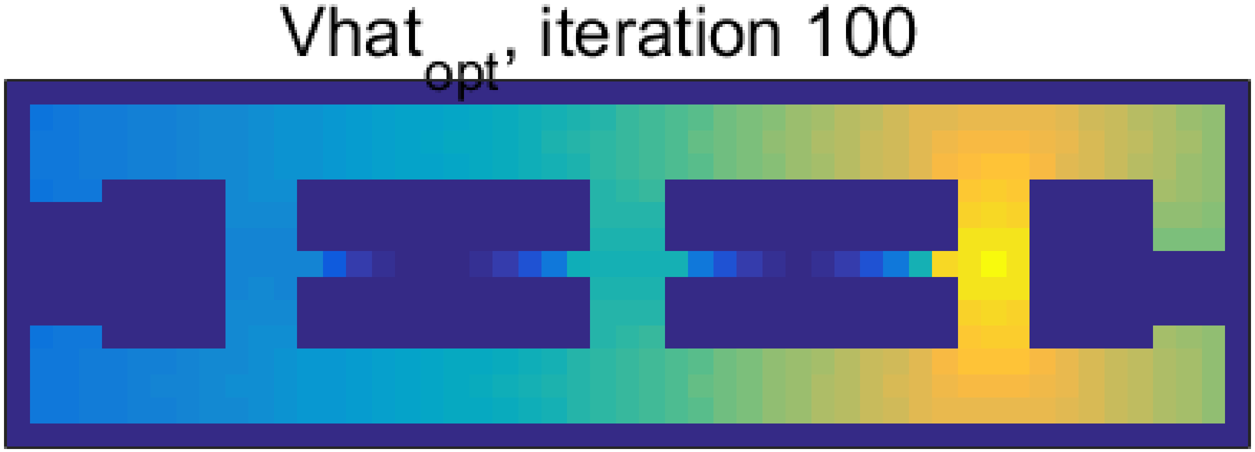

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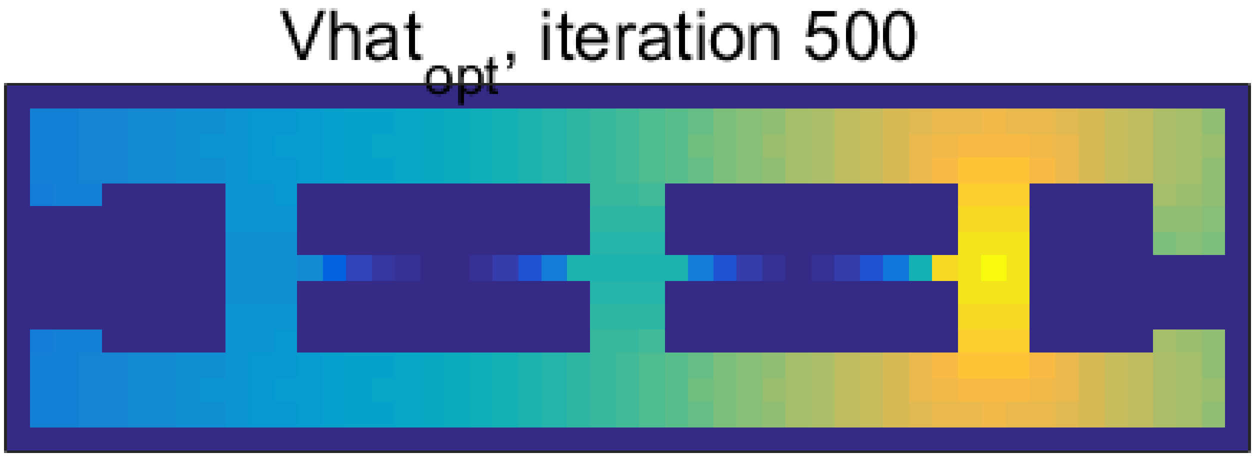

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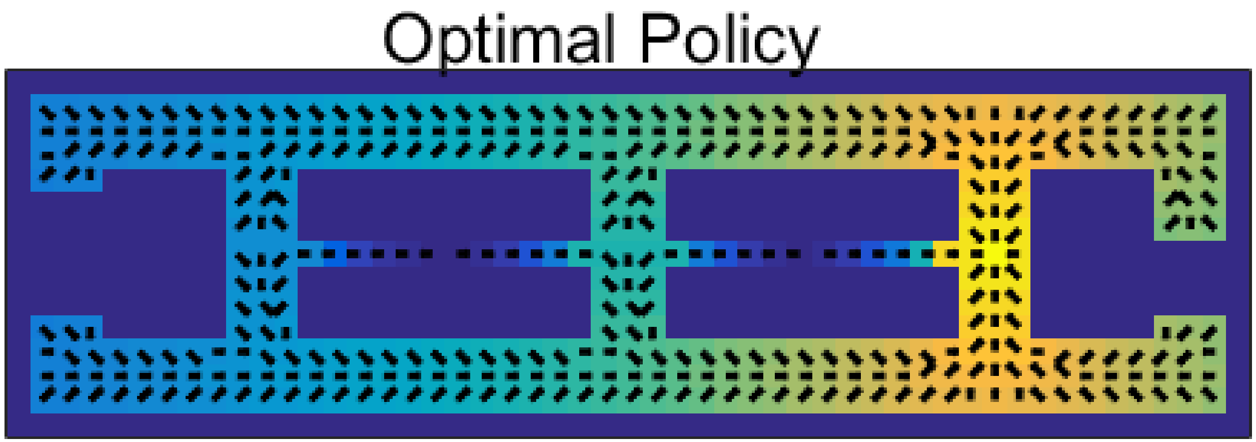

POLICY ITERATION

Greedy policy with respect to V:

$$(GV)(x) = \arg\max_a \left\{ r(x,a) + \sum_y P(y|x,a)V(x) \right\}$$

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Recall: $\pi^* = GV^*$

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THE CONVERGENCE OF ~~VALUE~~ policy ITERATION: PROOF SKETCH

Just replace T^* with the operator
$B^*: V \mapsto \left(T^{G(V)}\right)^{\infty}$

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THIS SHORT COURSE: A PRIMAL-DUAL VIEW

part 1

- Markov decision processes
  - Value functions and optimal policies
- Primal view: Dynamic programming
  - Policy evaluation, value and policy iteration
  - Value-function-based methods
    - Temporal differences, Q-learning, LSTD, deep Q networks,…

part 2

- Dual view: Linear programming
  - LP duality in MDPs
  - Direct policy optimization methods
    - Policy gradients, REPS, TRPO,…

FROM DYNAMIC PROGRAMMING TO VALUE-BASED REINFORCEMENT LEARNING

Policy iteration:

$$V_k$$

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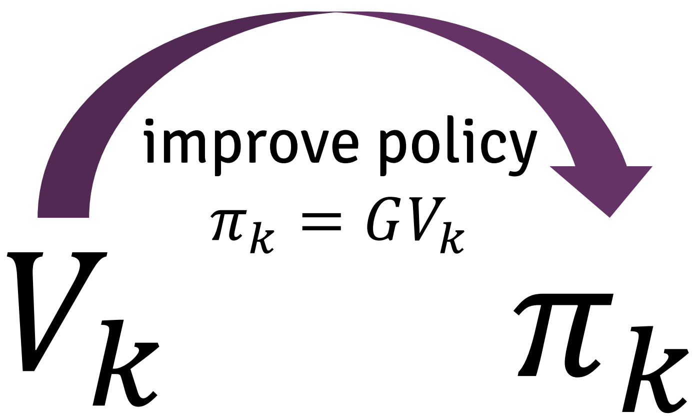

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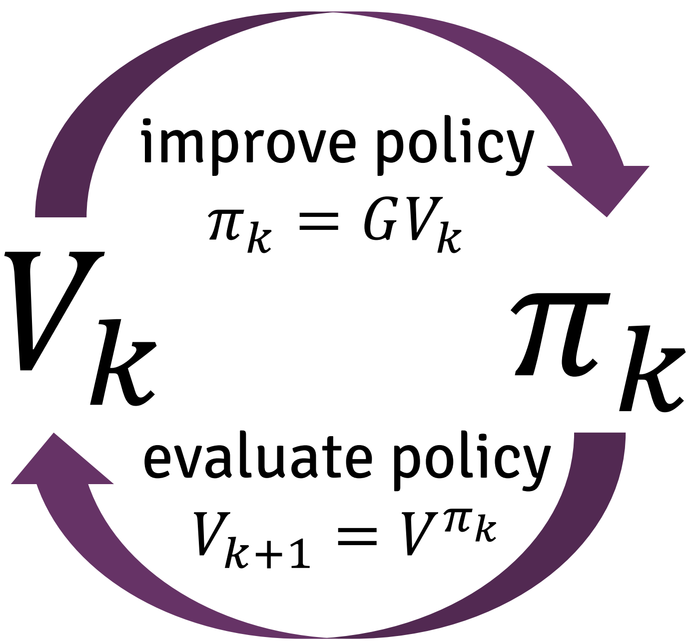

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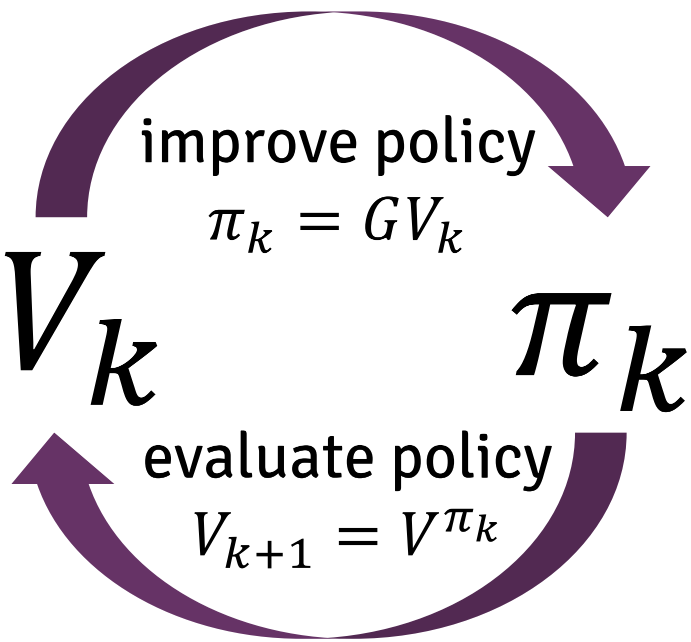
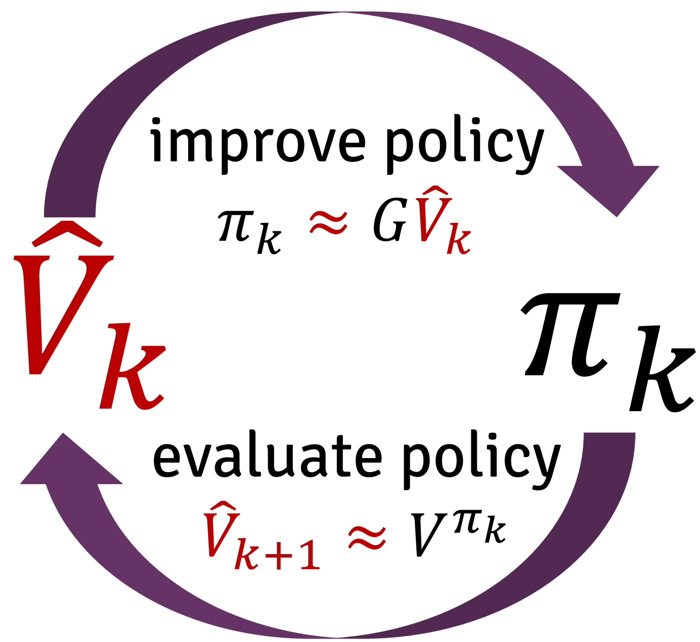


Approximate policy iteration:



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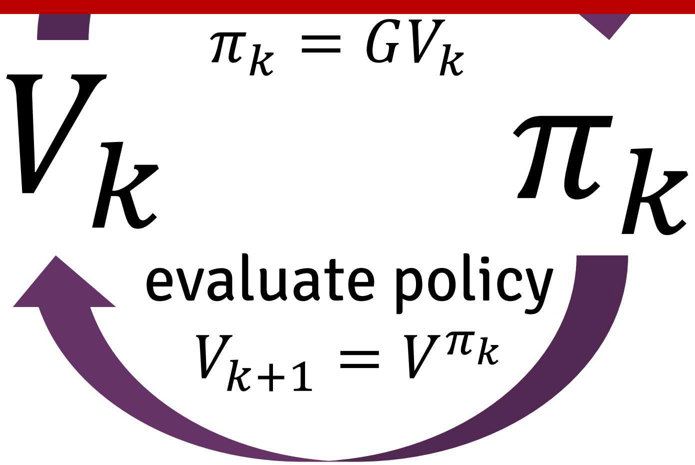
Fundamental RL tasks:

- Policy evaluation
- Policy improvement



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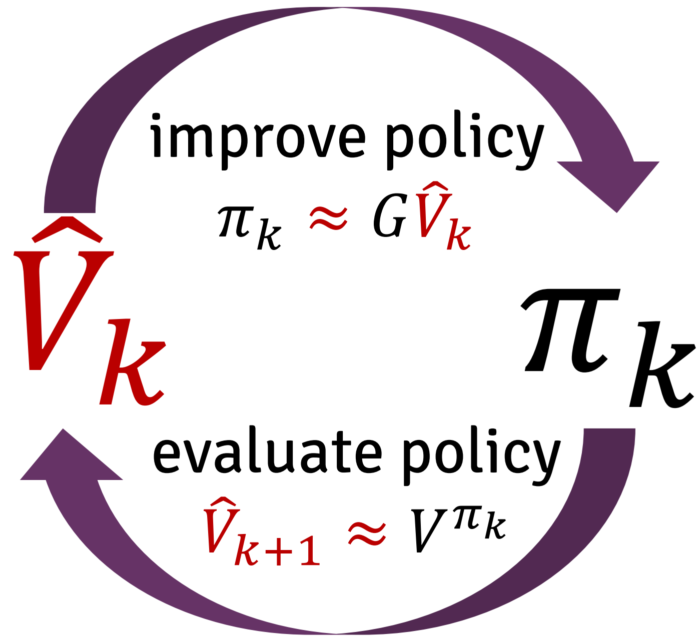

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Fundamental RL tasks:

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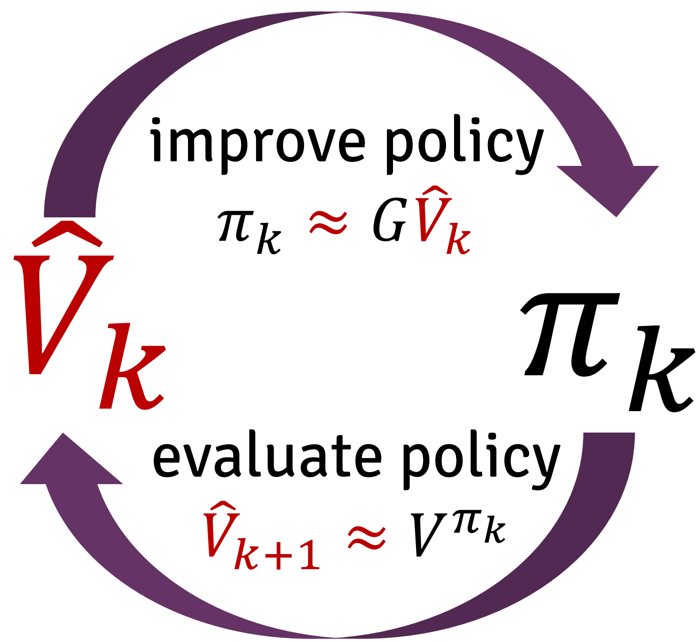
Challenges in RL:

- Unknown transition and reward functions ⇒ have to learn from sample access only
- State/action space can be large ⇒ V^* and π^* cannot be stored in memory

Approximate policy iteration:

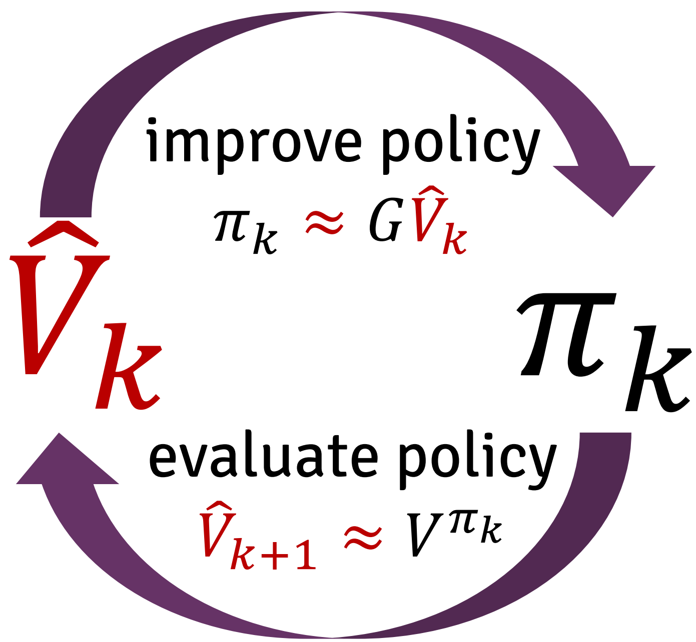


FROM DYNAMIC PROGRAMMING TO VALUE-BASED REINFORCEMENT LEARNING

Fundamental RL tasks:
- Policy evaluation
- Policy improvement

Challenges in RL:
- Unknown transition and reward functions ⇒ have to learn from sample access only
- State/action space can be large ⇒ V^* and π^* cannot be stored in memory

Approximate policy iteration:



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Full sample access to $P(\cdot\,|x,a)$ for any (x,a)

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DEALING WITH LARGE STATE SPACES

Idea: approximate V^* and/or π^* in a computationally tractable way!

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Approximating V^*: linear function approximation

- Define a set of d features:
$$\phi_i : X \to \mathbf{R}$$
- Parametrize value functions as
$$V_\theta(x) = \theta^\top \phi(x)$$
- Learning $V^* \Leftrightarrow$ Learning a good θ_*
$$V_{\theta^*} \approx V^*$$

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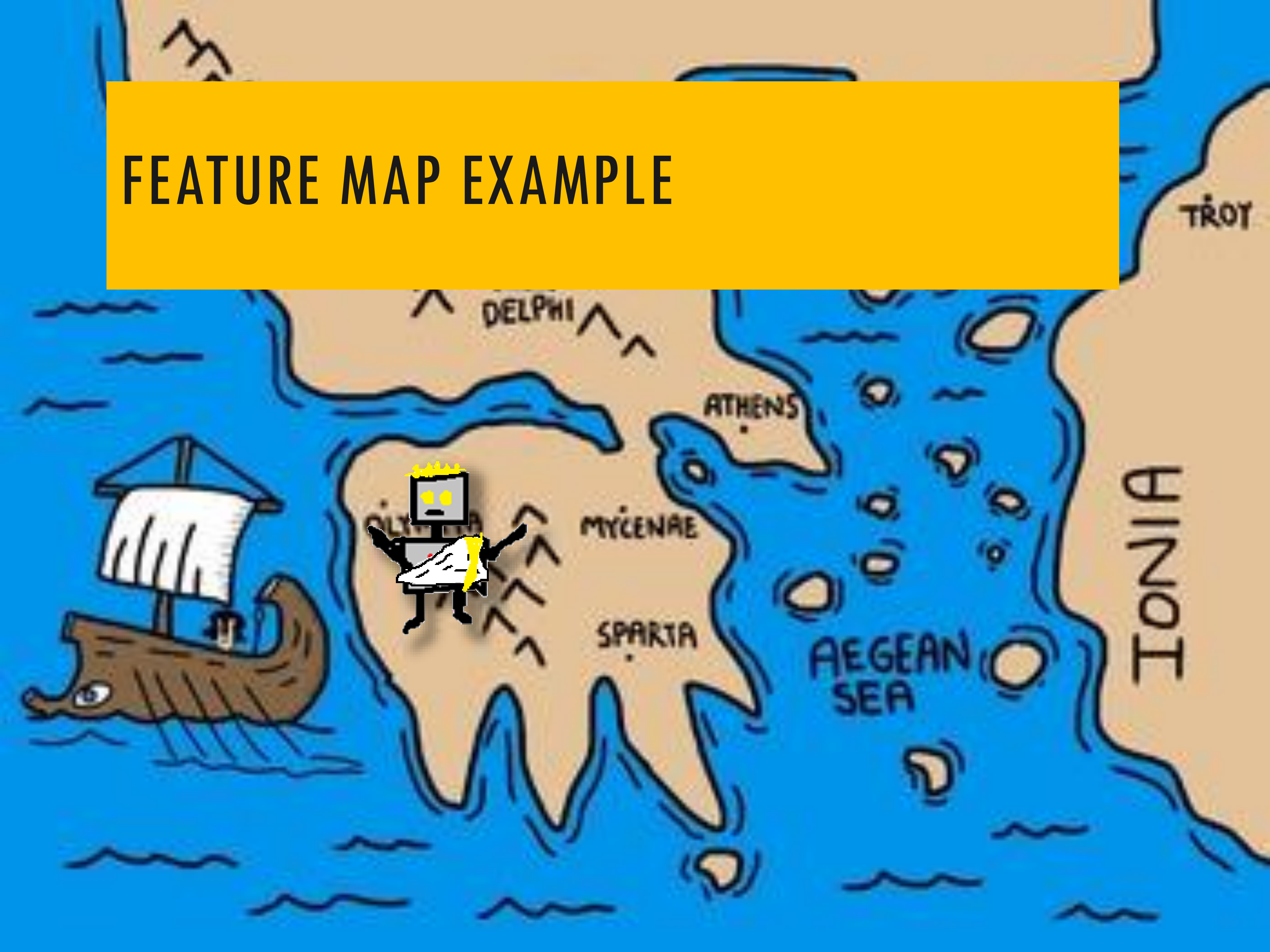

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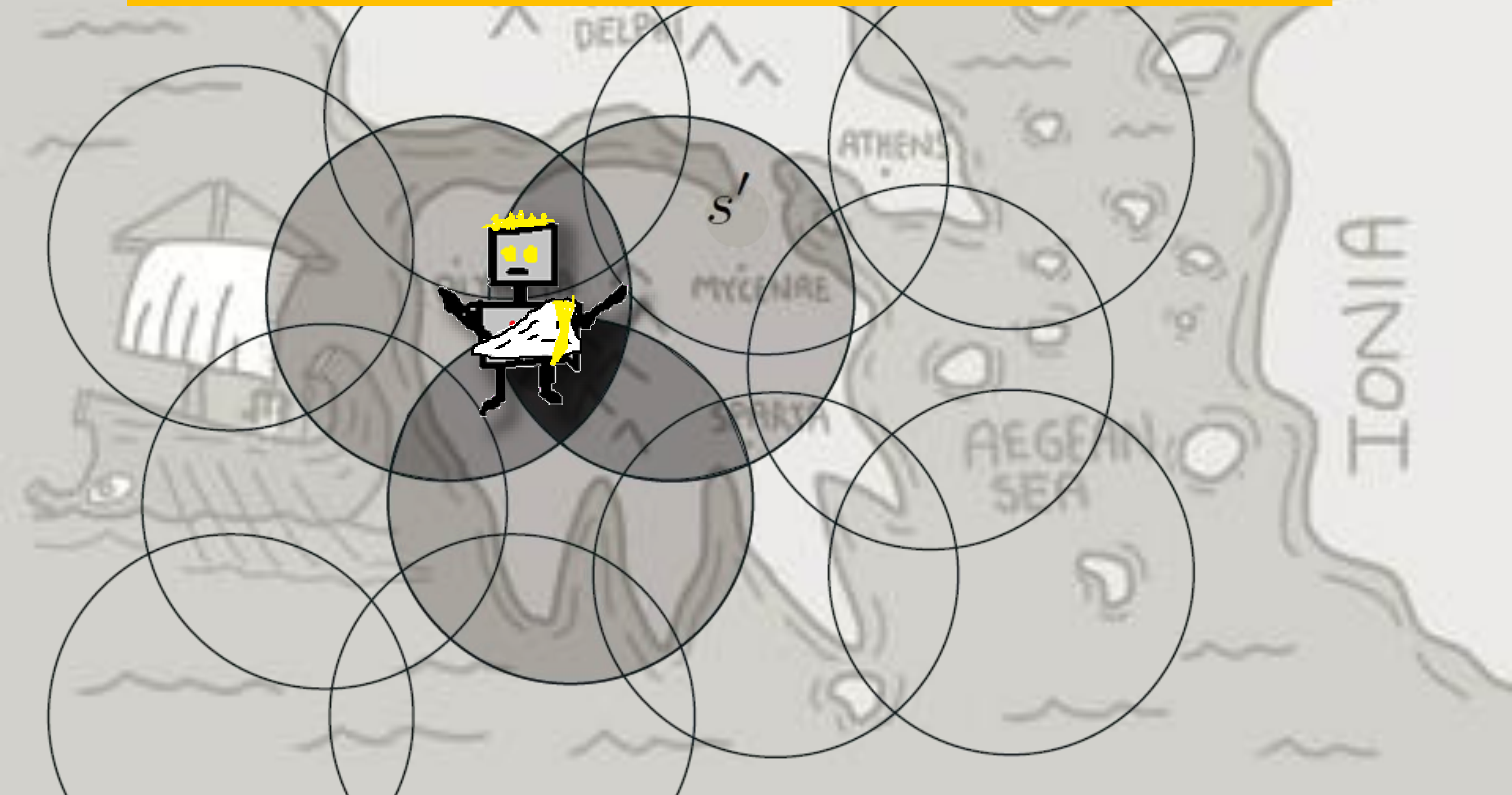

FEATURE MAP EXAMPLE

"coarse coding"

≈

indicator features

$\phi_i(x) = \mathbf{1}\{x \in X_i\}$

"PROST" FEATURES FOR ATARI GAMES

High-dimensional observations: 192×160 pixels

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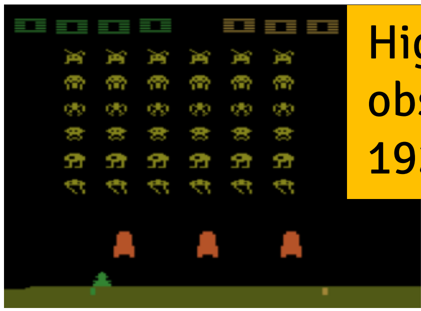

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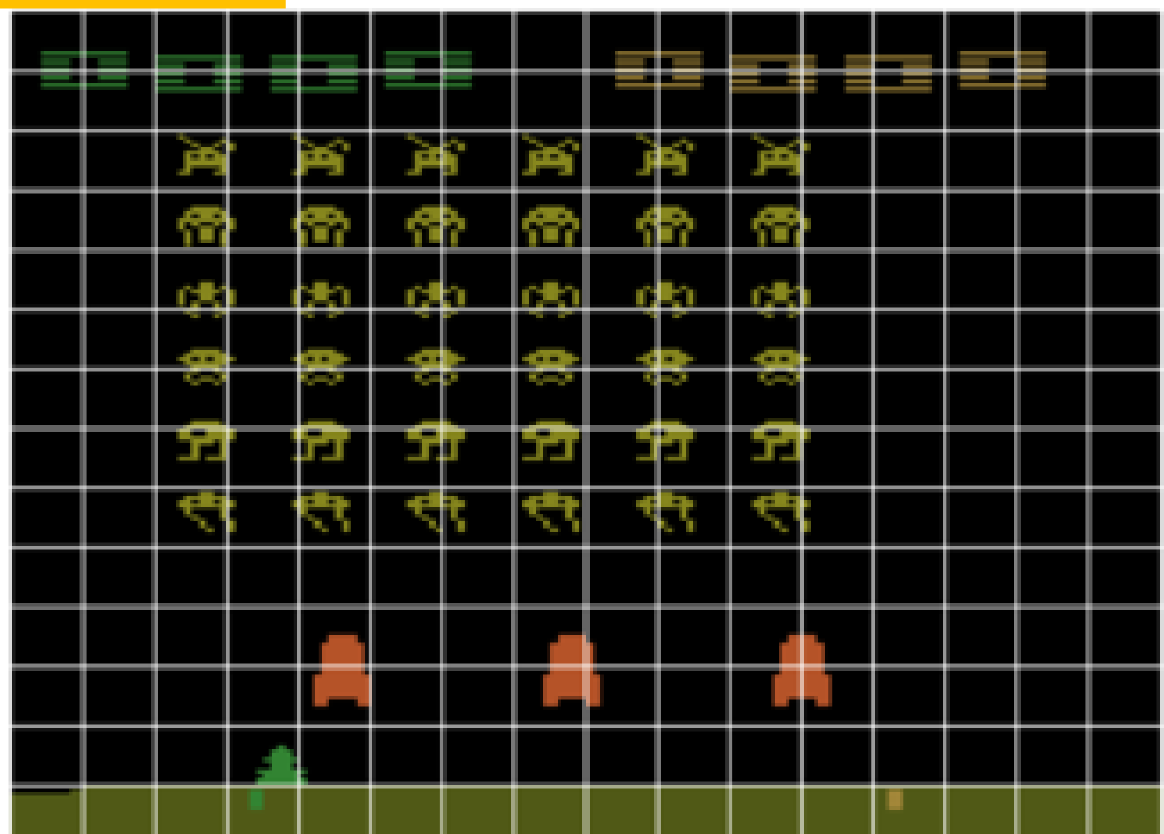

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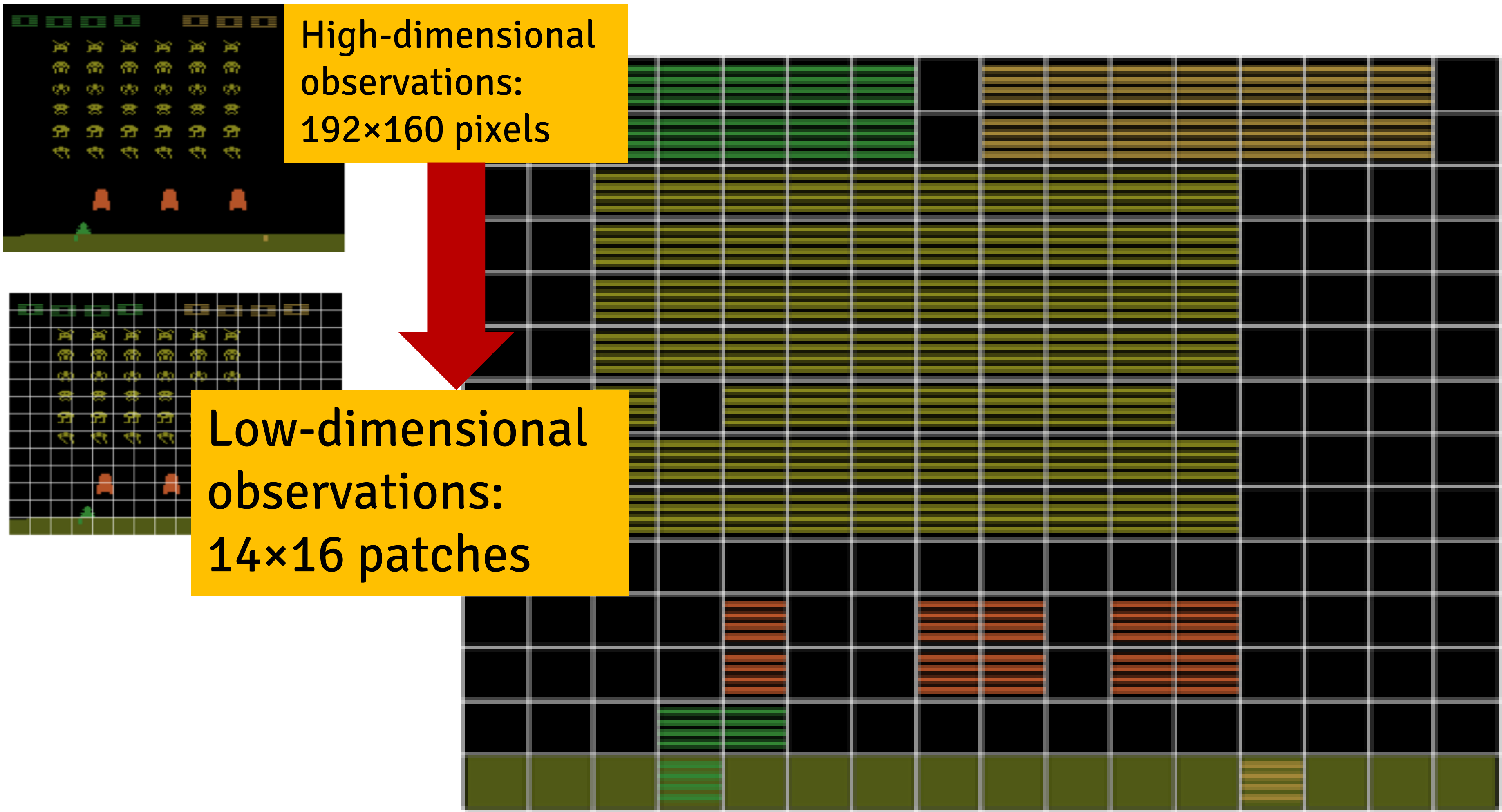

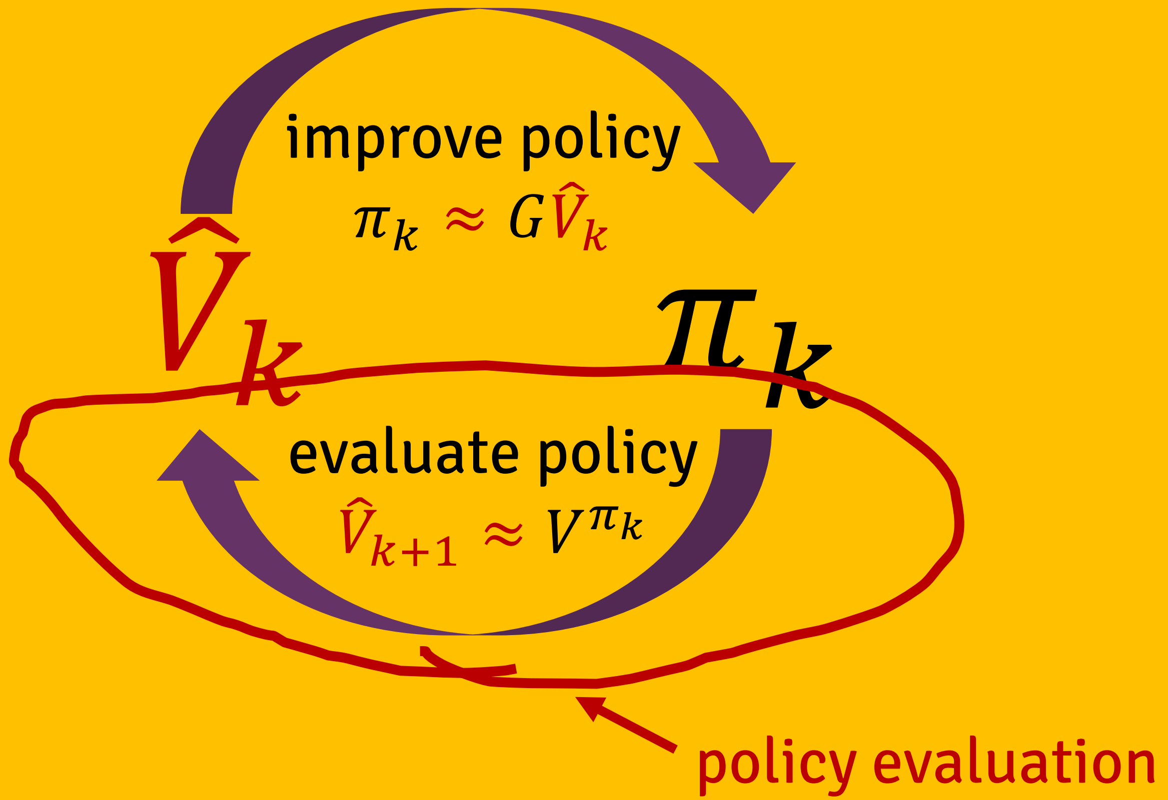

METHODS FOR POLICY EVALUATION

A GENTLE START: MONTE CARLO

Observe:

Policy evaluation = estimating V^{π}:

$V^{\pi}(x) = \mathbf{E}_{\pi}[\sum_{t=0}^{\infty} \gamma^{t} r(x_t, a_t) \,|x_0 = x]$

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Idea:
approximate $\mathbf{E}_\pi[\cdot]$ by sample averages!

- Simulate N trajectories using policy π
- For every state x that appears in the trajectories, let

$$\hat{V}_N(x) = \text{avg}\big(R_{1:N}(x)\big)$$

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Average of i.i.d. random variables:

$$\lim_{N\to\infty} \hat{V}_N = V^\pi$$

Collection of discounted returns $\sum_{t=0}^{T'} \gamma^t r_t$ after first visit to x

MONTE CARLO WITH FEATURES

Monte Carlo policy evaluation

Input:

N trajectories $\sim \pi$, feature map $\phi : X \to \mathbb{R}^d$

Output:

$$\hat{V}_N = \arg\min_{\theta \in \mathbb{R}^d} \mathbf{E}_x \left[\left(\theta^\top \phi(x) - R_{1:N}(x) \right)^2 \right]$$

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Least-squares fit of discounted returns

PROPERTIES OF MONTE CARLO

☺ Value estimates converge to true values ☺

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Idea: construct an objective that uses the Bellman equations

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The Bellman error

$$L(V) = \mathbf{E}_{x\sim\mu}\left[\left(T^\pi V(x) - V(x)\right)^2\right]$$

TEMPORAL DIFFERENCE LEARNING

Idea: use stochastic approximation to reduce instantaneous Bellman error

$$\Delta_t = \left(T^{\pi}\hat{V}_t(x_t) - \hat{V}_t(x_t)\right)^2$$

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TD(0)

Input: arbitrary function $\hat{V}_0 : X \to \mathbf{R}$

For $t = 0,1, \ldots,$

$$\delta_t = r_t + \gamma \hat{V}_t(x_{t+1}) - \hat{V}_t(x_t)$$

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Converges if step-sizes satisfy

$\sum_{t=0}^{\infty} \alpha_t = \infty$ and $\sum_{t=0}^{\infty} \alpha_t^2 < \infty$

(e.g., $\alpha_t = c/t$ does the job)

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In equilibrium,

$$\mathbf{E}\left[r_t + \gamma \hat{V}_t(x_{t+1}) - \hat{V}_t(x_t)\right] = 0$$

TD(0) WITH LINEAR FUNCTION APPROXIMATION

Let $\phi: X \to \mathbf{R}^d$ be a feature vector

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Approximating $V^\pi(x) \approx \theta^\top \phi(x)$ by TD(0):

TD(0) with LFA

Input: arbitrary param. vector $\theta_0 \in \mathbf{R}^d$

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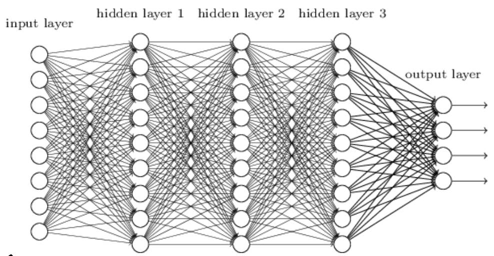
This still converges to V^π!!!

OK, well, somewhere nearby…

TD(0) WITH NONLINEAR FUNCTION APPROXIMATION

Let $V_\theta: X \to R$ be a parametric class of functions (e.g., deep neural network)



Approximating $V^\pi(x) \approx V_\theta(x)$ by TD(0):

TD(0) with general FA

Input: arbitrary param. vector $\theta_0 \in \mathbf{R}^d$

For $t = 0,1, \dots,$

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Let $V_\theta : X \to R$ be a p… functions (e.g., deep…

Approximating $V^\pi(x) \approx V_\theta(x)$ by TD(0):

Not much is known about convergence ☹

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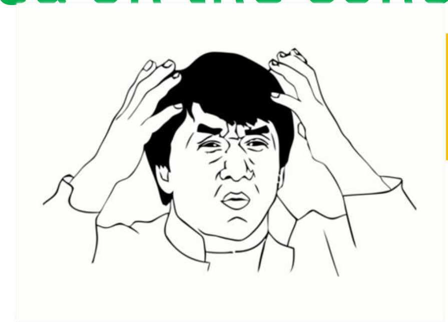

= "bootstrapping"

WHERE DOES TD(0) CONVERGE TO?

TD(0) with LFA

Input: arbitrary param. vector $\theta_0 \in \mathbf{R}^d$

For $t = 0,1, \dots,$

$$\delta_t(\theta) = r_t + \gamma\theta^\top\phi(x_{t+1}) - \theta^\top\phi(x_t)$$

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In the limit, TD(0) finds a θ^* such that

$$\mathbf{E}[\delta_t(\theta^*)\phi(x_t)] = 0$$

WHERE DOES TD(0) CONVERGE TO?

Idea: given a finite trajectory, approximate the TD fixed point by solving

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This is a linear system

$$A_T \theta = b_T$$

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A_T $\qquad$ b_T

LEAST-SQUARES TEMPORAL DIFFERENCE LEARNING (LSTD)

LSTD(0)

Input: trajectory $(x_t, a_t, r_t)_{t=1}^{T}$

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☹ computational complexity: $O(Td^2 + d^3)$ ☹

☹ A_T^{-1} may not exist for small T ☹

TD(0):
$O(Td)$

THE CONVERGENCE OF TD(0) AND LSTD(0)

Theorem

In the limit $T \to \infty$, LSTD(0) and TD(0) both minimize the projected Bellman error

$$L(V) = \mathbf{E}_{x\sim\mu}\left[\left(\Pi_{\phi}[T^{\pi}V(x)] - V(x)\right)^{2}\right]$$

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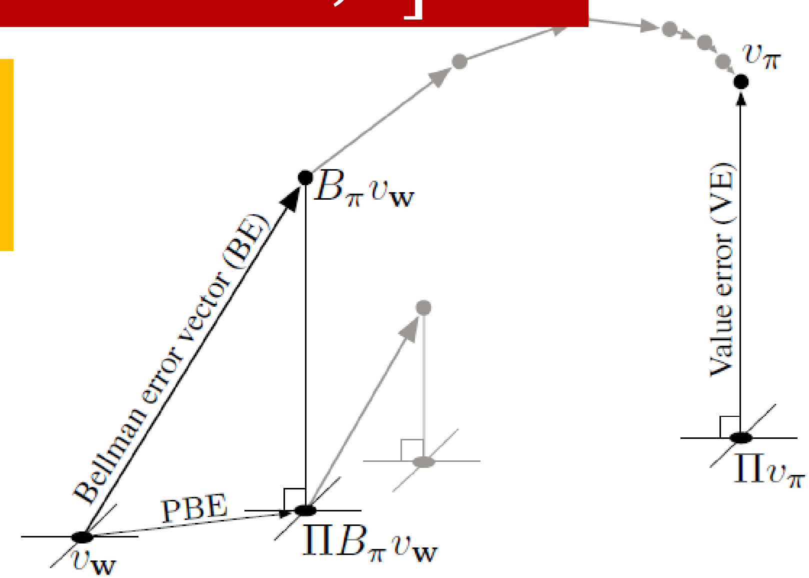
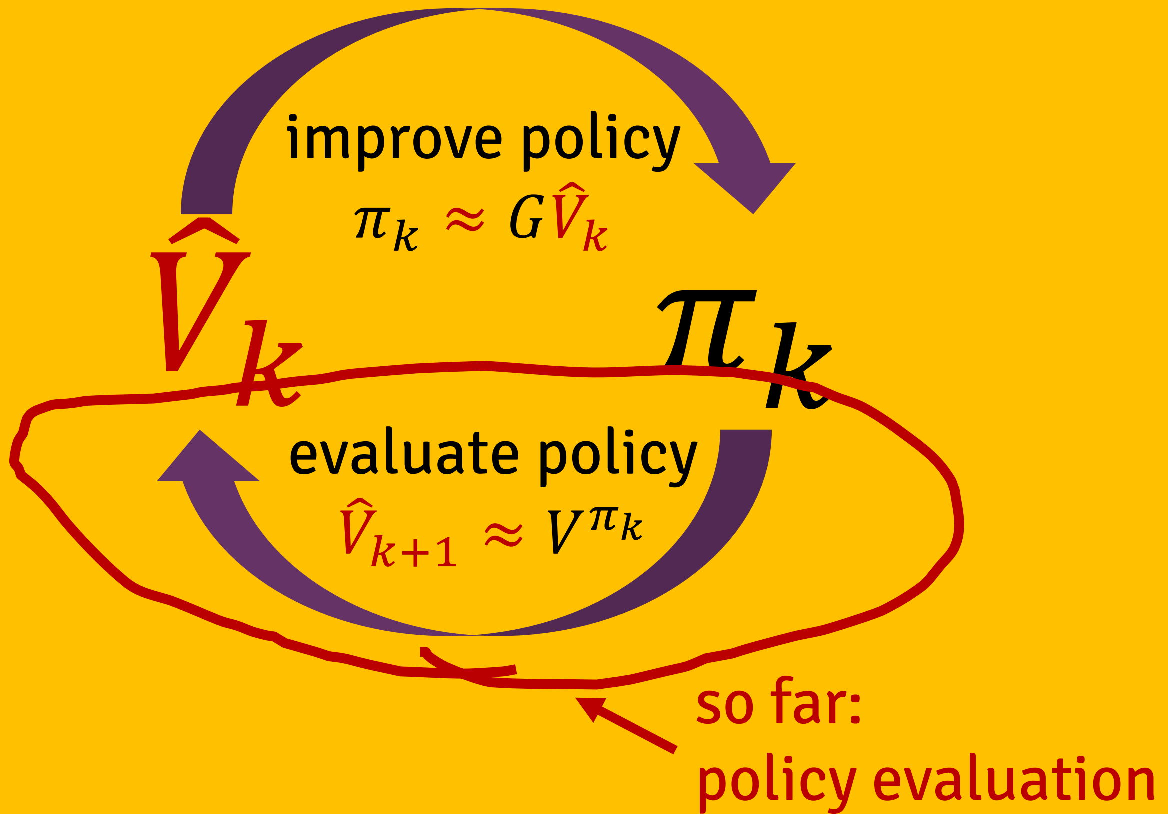
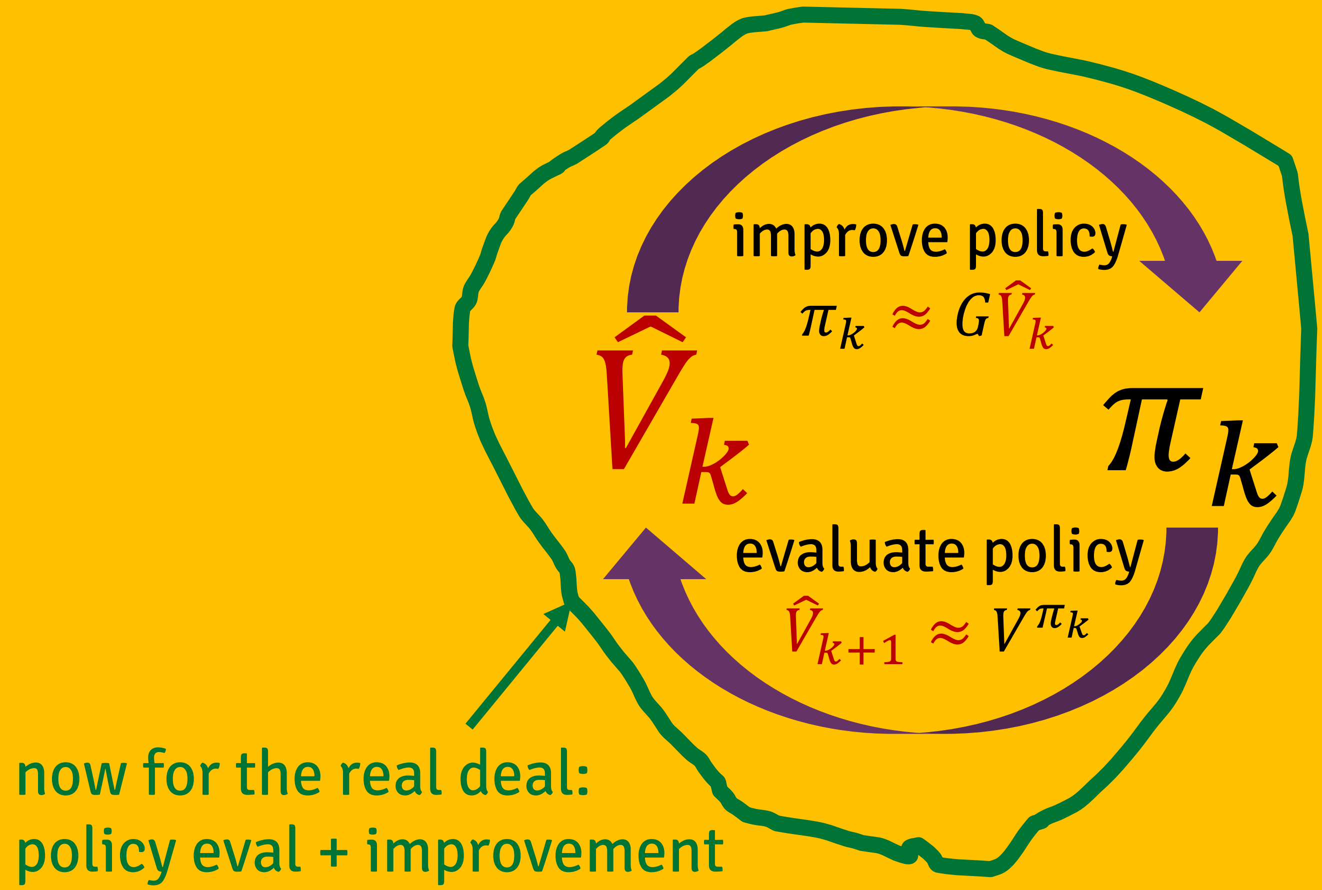
Projection onto span of features





FROM POLICY EVALUATION POLICY IMPROVEMENT



FROM POLICY EVALUATION POLICY IMPROVEMENT

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- Compute ε-greedy policy w.r.t. $\hat{Q}_t$:

$$\pi_t(x) = \begin{cases} \arg\max \hat{Q}_t(x,a)\,, & \text{w.p. } 1-\varepsilon \\ \text{uniform random action}, & \text{w.p. } \varepsilon \end{cases}$$

- Improve estimated $\hat{Q}_{t+1}$ by reducing Bellman error

$$\Delta_t = \left(\mathbf{E}\left[r_t + \gamma \max_a \hat{Q}_t(x_{t+1},a)\right] - \hat{Q}_t(x_t,a_t)\right)^2$$

OFF-POLICY CONTROL: Q-LEARNING

Off-policy learning: evaluating π^* while following suboptimal policy!

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OFF-POLICY CONTROL: Q-LEARNING

Q-learning

Input: arbitrary $\hat{Q}_0 : X \times A \to \mathbf{R}$

For $t = 0,1, \dots,$

- Choose action $a_t \sim \varepsilon$-greedy w.r.t. $\hat{Q}_t$
- Observe r_t, x_{t+1}
- Compute

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ON-POLICY CONTROL: SARSA

SARSA

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SARSA = $(s_t, a_t, r_t, s_{t+1}, a'_{t+1})$

ON-POLICY CONTROL: SARSA

SARSA ~ XARXA

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- Observe r_t, x_{t+1}, a'_{t+1}
- Compute

  $\delta_t = r_t + \gamma \hat{Q}_t(x_{t+1}, a'_{t+1}) - \hat{Q}_t(x_t, a_t)$

  $\hat{Q}_{t+1}(x_t, a_t) = \hat{Q}_t(x_t, a_t) + \alpha_t \delta_t$

$a'_{t+1} \sim \varepsilon$ −greedy: on-policy

SARSA = $(s_t, a_t, r_t, s_{t+1}, a'_{t+1})$

Q-LEARNING VS. SARSA WITH FUNCTION APPROXIMATION

Both algorithms can be adapted to linear and non-linear FA by using the update rule

$$\theta_{t+1} = \theta_t + \alpha_t \delta_t \nabla_\theta Q_\theta(x_t, a_t)$$

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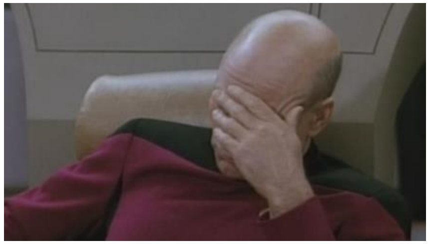
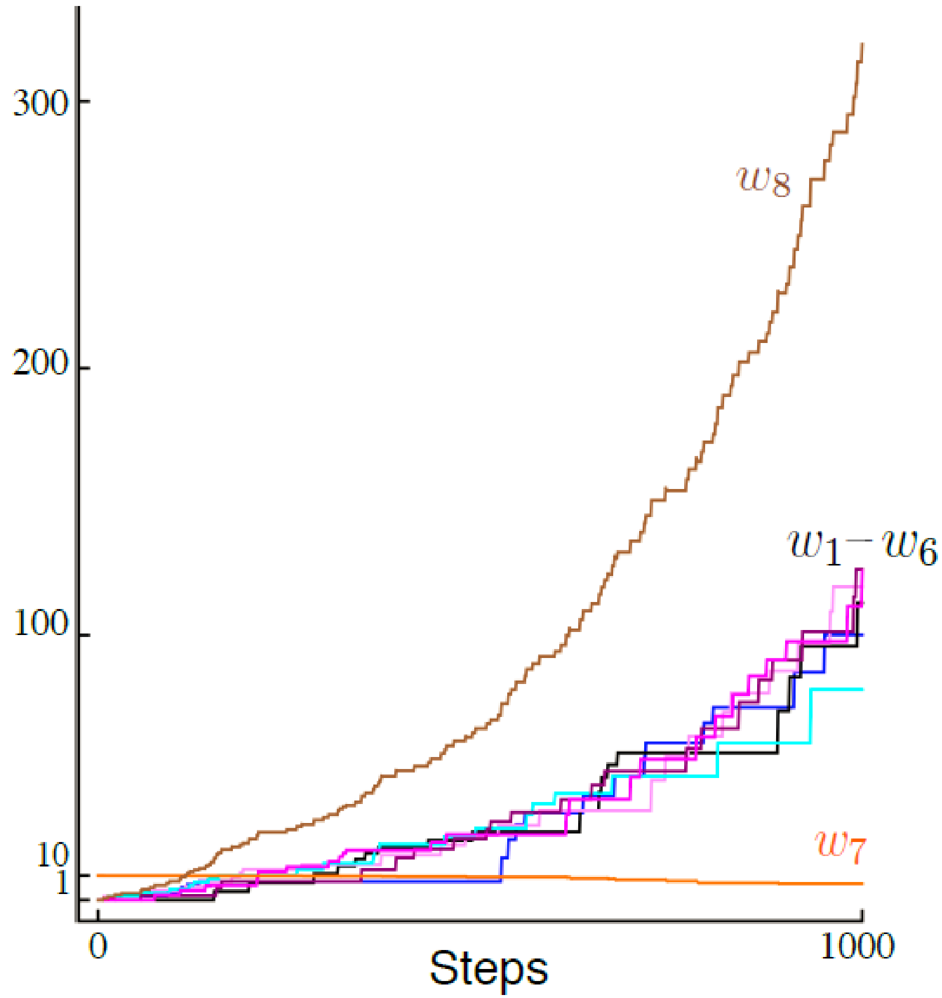
  - Proposed fixes: gradient TD algorithms, emphatic TD algorithms, double Q-learning, soft Q-learning, G-learning,…
  - Practical solution: tune it until it works



DIVERGENCE OF OFF-POLICY TD LEARNING

The "deadly triad":

- Function approximation
- Bootstrapping
- Off-policy learning



DIVERGENCE OF OFF-POLICY TD LEARNING

The "deadly triad":

- Function approximation
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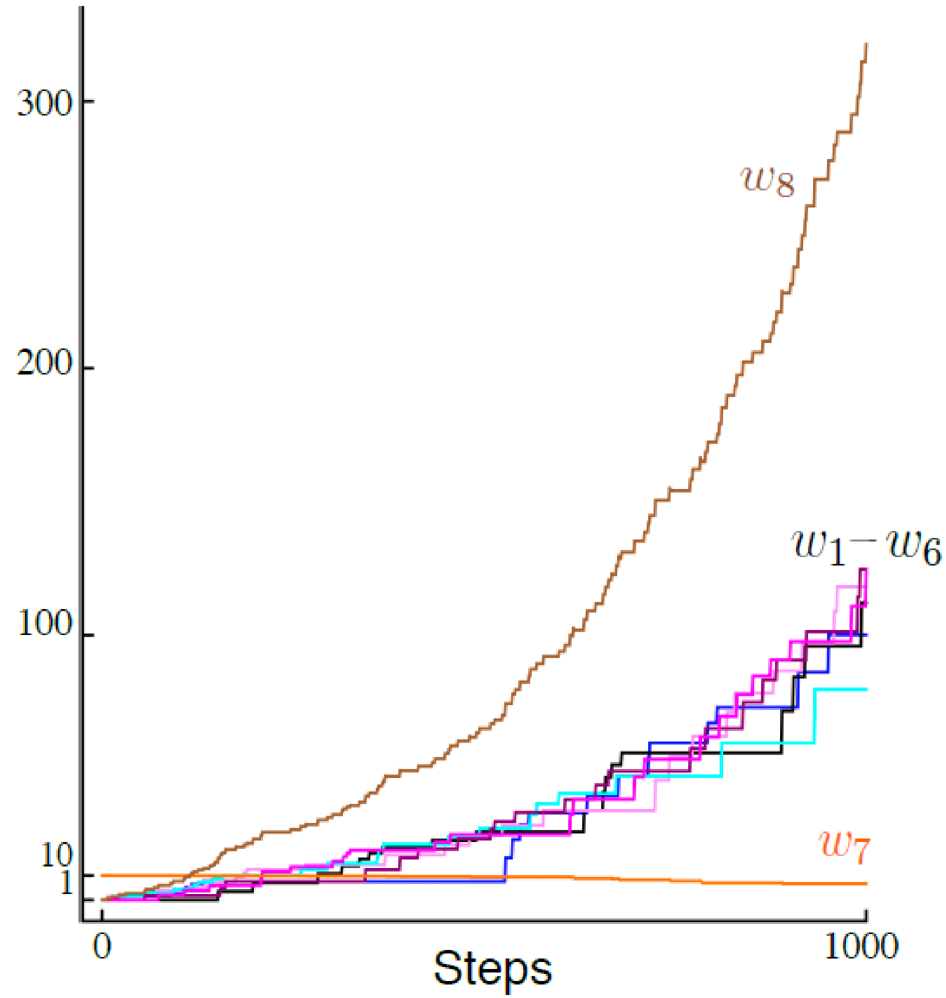
BUT

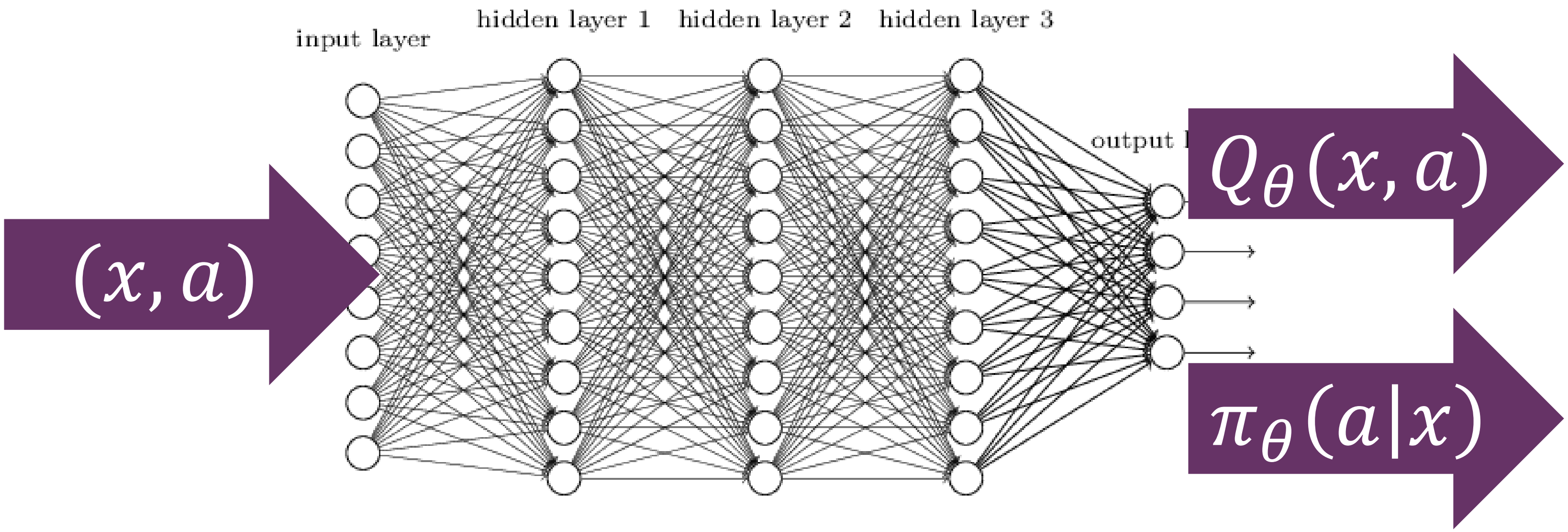
Divergence is typically not too extreme when behavior policy is close to evaluation policy and FA is linear



DEEP REINFORCEMENT LEARNING

THE PROMISE OF DEEP REINFORCEMENT LEARNING

Parametrize Q-function/policy by a deep net



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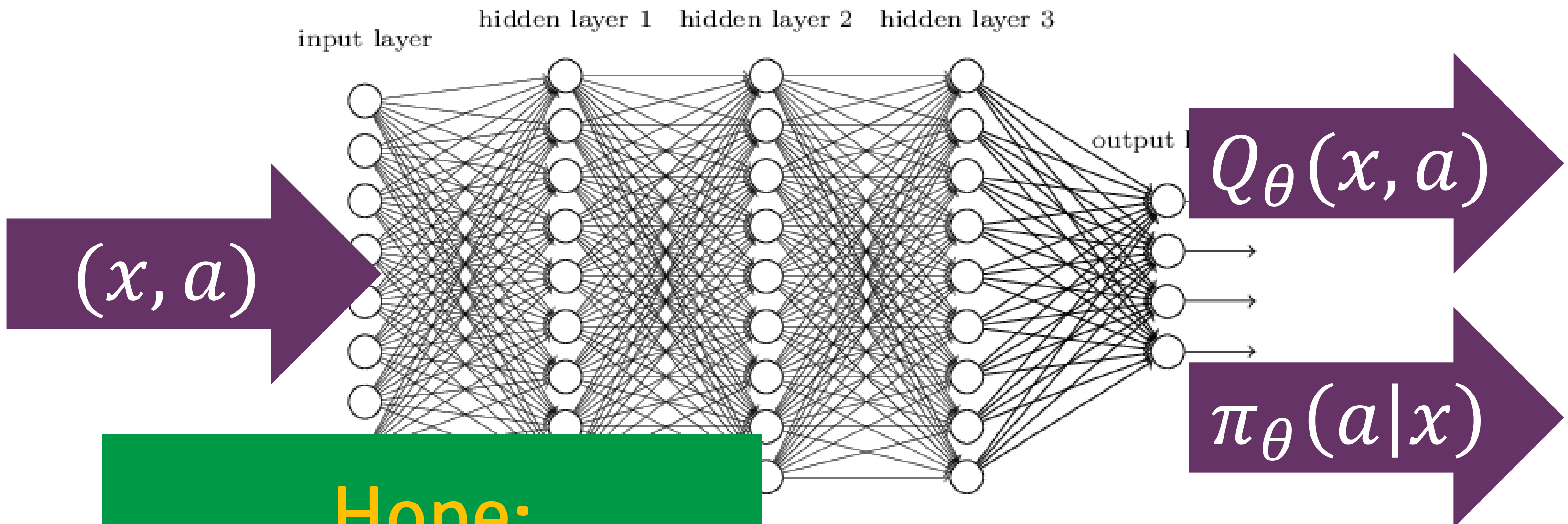

Hope:

Take advantage of representation power!

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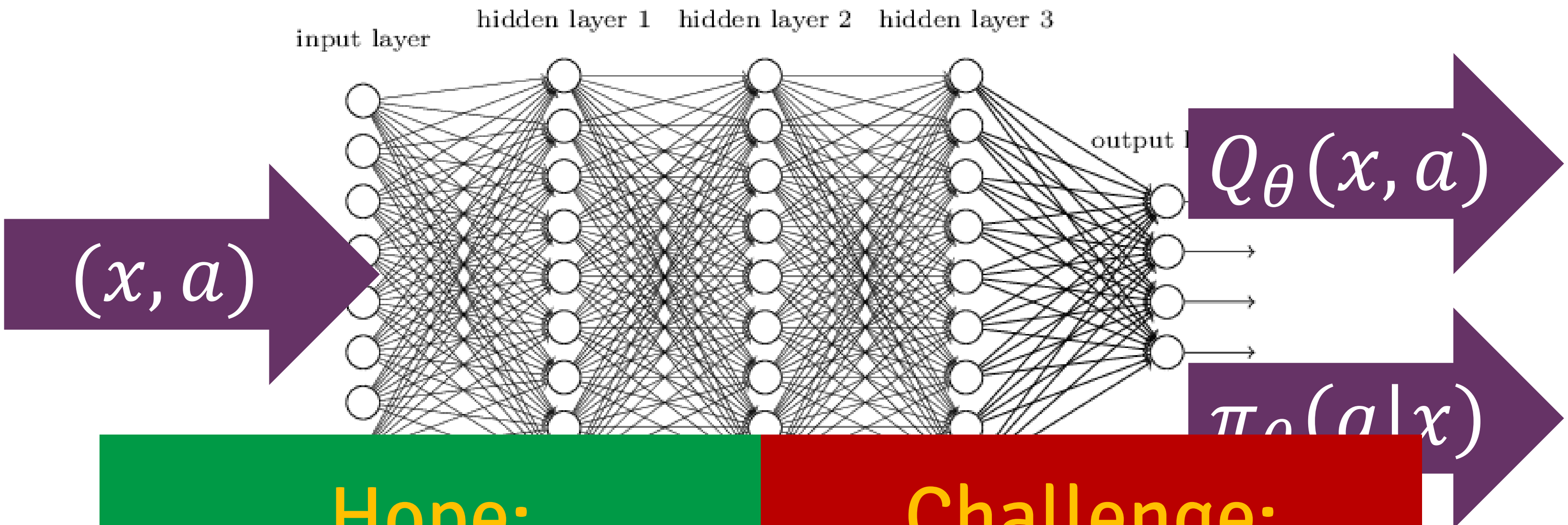

Hope:
Take advantage of representation power!

Challenge:
Existing RL methods difficult to generalize

LEAST-SQUARES TEMPORAL DIFFERENCE LEARNING (LSTD)

LSTD(0)

Input: trajectory $(x_t, a_t, r_t)_{t=1}^{T}$

$$\theta_T = A_T^{-1} b_T$$
$$\hat{V}_T = \theta_T^{\top} \phi$$

☹☹☹

Idea not directly applicable to non-linear function approximation!

☹☹☹

LSTD FOR NON-LINEAR FUNCTION APPROXIMATION?

Can we optimize Bellman error

$$L(\theta) = \mathbf{E}_{x\sim\mu}\left[\left(T^{\pi}V_{\theta}(x) - V_{\theta}(x)\right)^{2}\right]$$

by stochastic gradient descent????

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NO!!

Bellman error involves a double expectation:

$$L(\theta) = \mathbf{E}_X[\ell(\theta; X, \mathbf{E}_Y[Y|X])]$$

can't get unbiased gradients!

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by stochastic g ?

The infamous
"double sampling"
issue of RL

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can't get unbiased gradients!

TACKLING DOUBLE SAMPLING

- Saddle-point optimization:

$$\min_{\theta} \mathbf{E}[f(\theta; X, \mathbf{E}[Y|X])^2]$$

TACKLING DOUBLE SAMPLING

- Saddle-point optimization:

$$\min_{\theta} \mathbf{E}[f(\theta; X, \mathbf{E}[Y|X])^2] =$$

$$\min_{\theta} \max_{z} \mathbf{E}[z(X,Y) \cdot f(\theta; X, \mathbf{E}[Y|X])] - \mathbf{E}[z^2(X,Y)]$$

TACKLING DOUBLE SAMPLING

No nested expectation here!

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$\Rightarrow$ "modified Bellman residual" (Antos et al. 2008), "Gradient TD" methods (Sutton et al. 2009), SBEED (Dai et al., 2018)

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- Iterative optimization schemes

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- Iterative optimization schemes

FITTED POLICY EVALUATION

Idea: compute sequence of value functions by minimizing

$$L_n(\hat{V}; \hat{V}_k) = \frac{1}{n}\sum_{t=1}^{n}\left(r_t + \hat{V}_k(x_{t+1}) - \hat{V}(x_t)\right)^2$$

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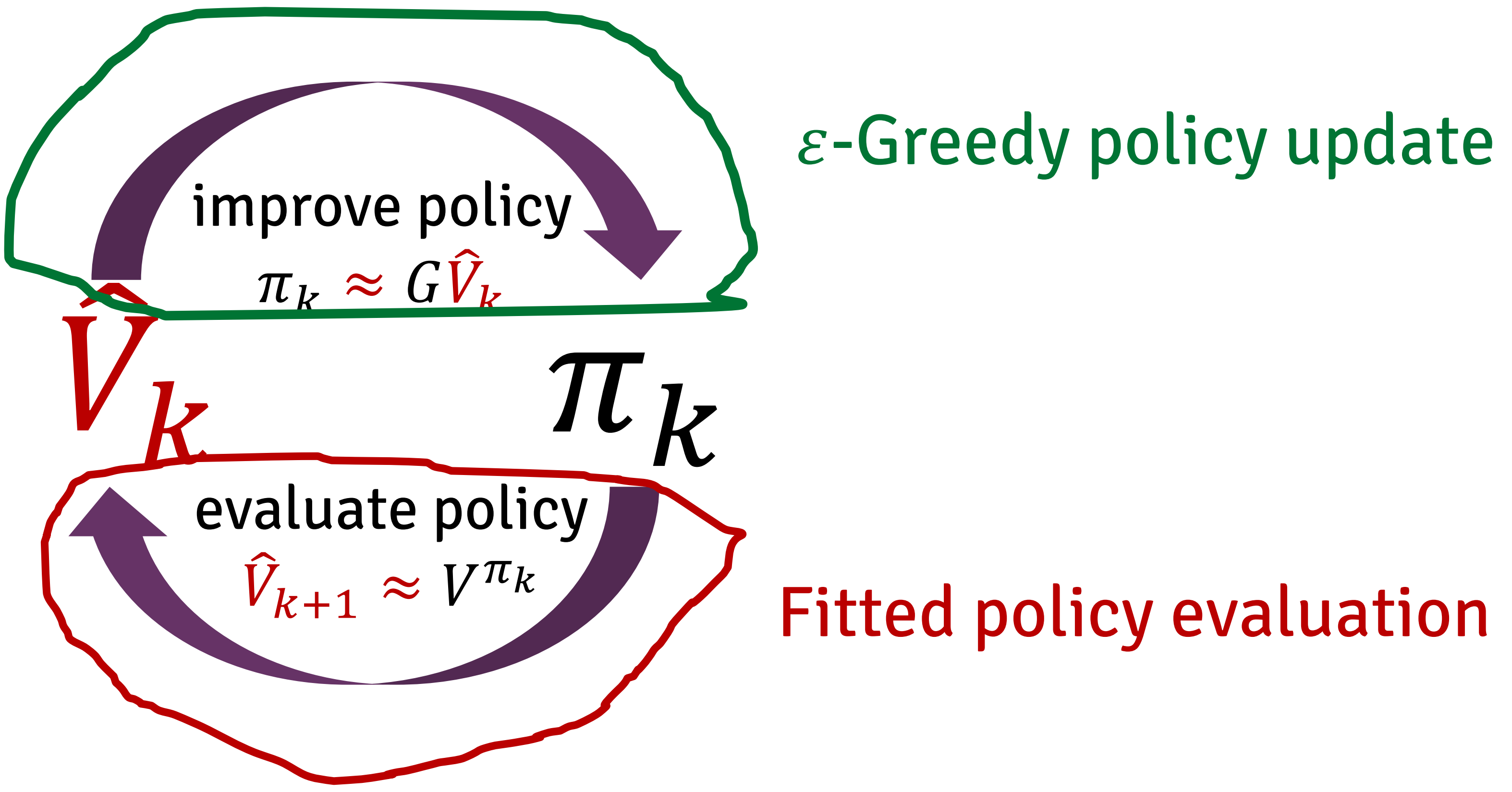
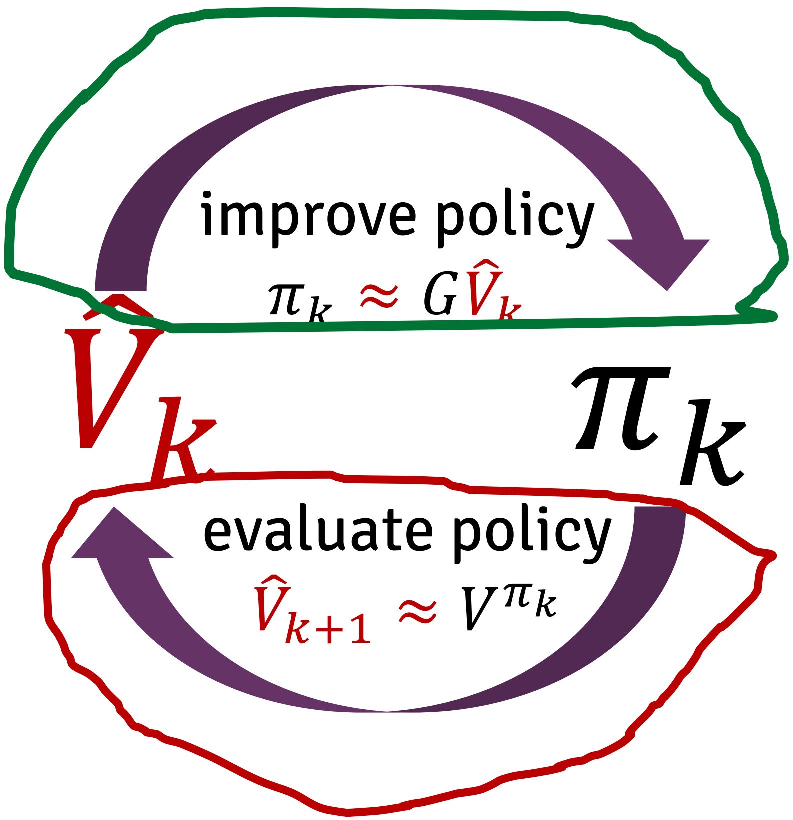
This can be finally treated as a regression problem & solved by SGD!

FITTED POLICY ITERATION



FITTED POLICY ITERATION



ε-Greedy policy update

Computing policy needs model of P… better use Q-functions!

Fitted policy evaluation

FITTED VALUE ITERATION

Idea: compute sequence of Q-value functions by minimizing

$$L_n(\hat{Q}; \hat{Q}_k) = \frac{1}{n}\sum_{t=1}^{n}\Big(\underbrace{r_t + \max_a \hat{Q}_k(x_{t+1}, a)}_{\text{Target}} - \underbrace{\hat{Q}(x_t, a_t)}_{\text{Free variable}}\Big)^2$$

FITTED VALUE ITERATION

Fitted value iteration

Input: function space F, $\hat{Q}_0 \in F$

For $k = 0,1, \ldots,$

- $\pi_k = G_\varepsilon \hat{Q}_k$
- generate trajectory
$$(x_t, a_t, r_t)_{t=1}^n \sim \pi_k$$
- compute
$$\hat{Q}_{k+1} = \underset{\hat{Q} \in F}{\operatorname{argmin}} L_n(\hat{Q}; \hat{Q}_k)$$

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Computing policy is trivial!

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- compute
$$\hat{Q}_{k+1} = \underset{\hat{Q} \in F}{\operatorname{argmin}}\, L_n\big(\hat{Q}; \hat{Q}_k\big)$$

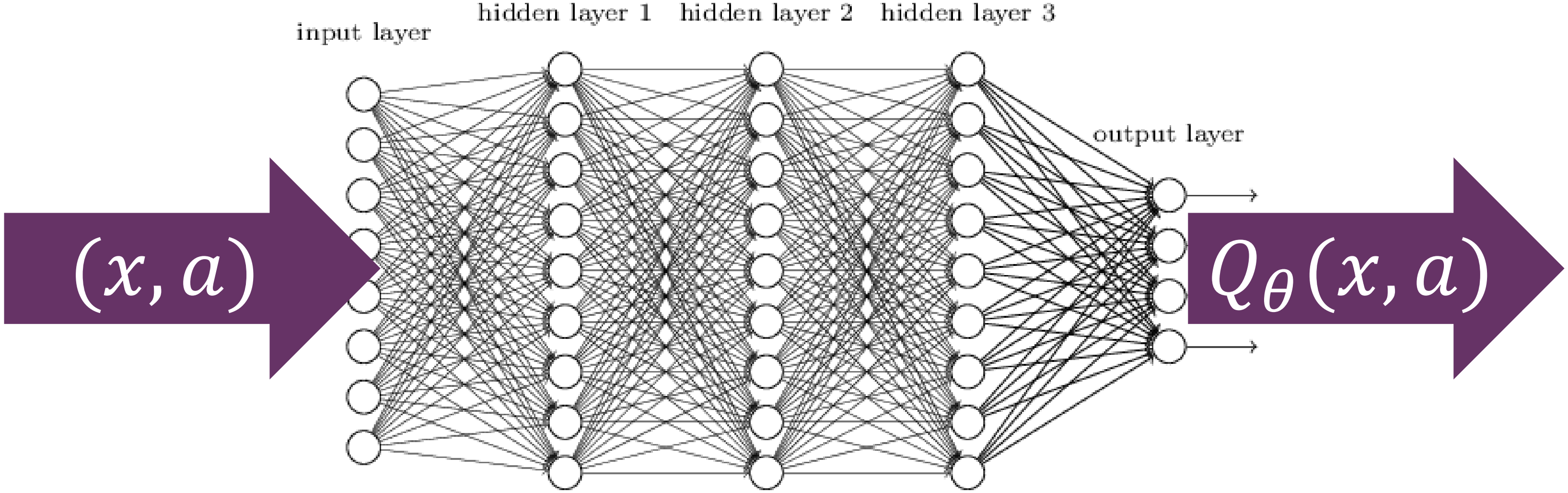
Computing policy is trivial!

Convergence can be guaranteed!

under very technical assumptions…

DEEP Q NETWORKS

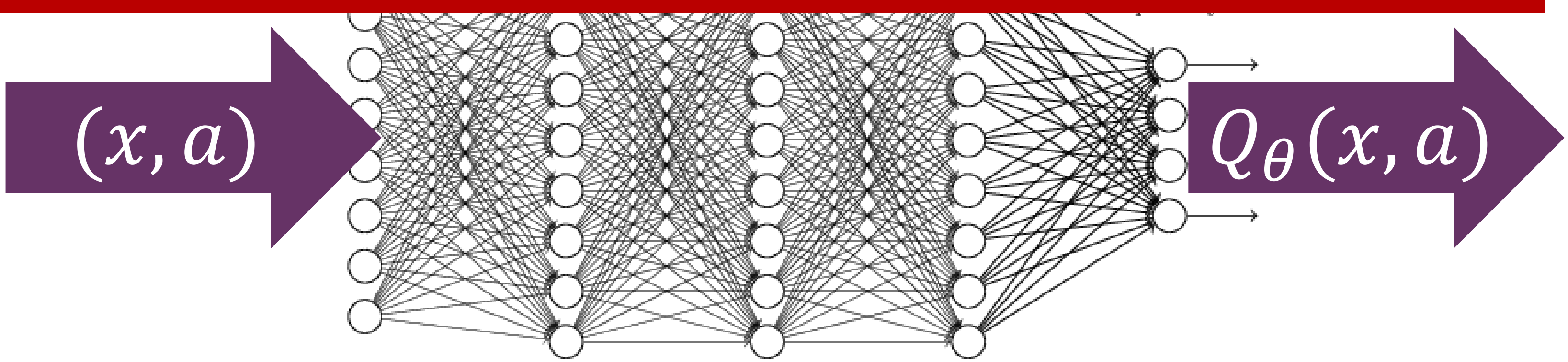
Parametrize Q-function by a deep neural net



DEEP Q NETWORKS

Minimize the loss

$$\mathbf{E}_{(X,A,R,X')\sim D}\left[\left(R+\gamma\max_{b} Q_{\theta_k}(X',b)-Q_\theta(X,A)\right)^2\right]$$



DEEP Q NETWORKS

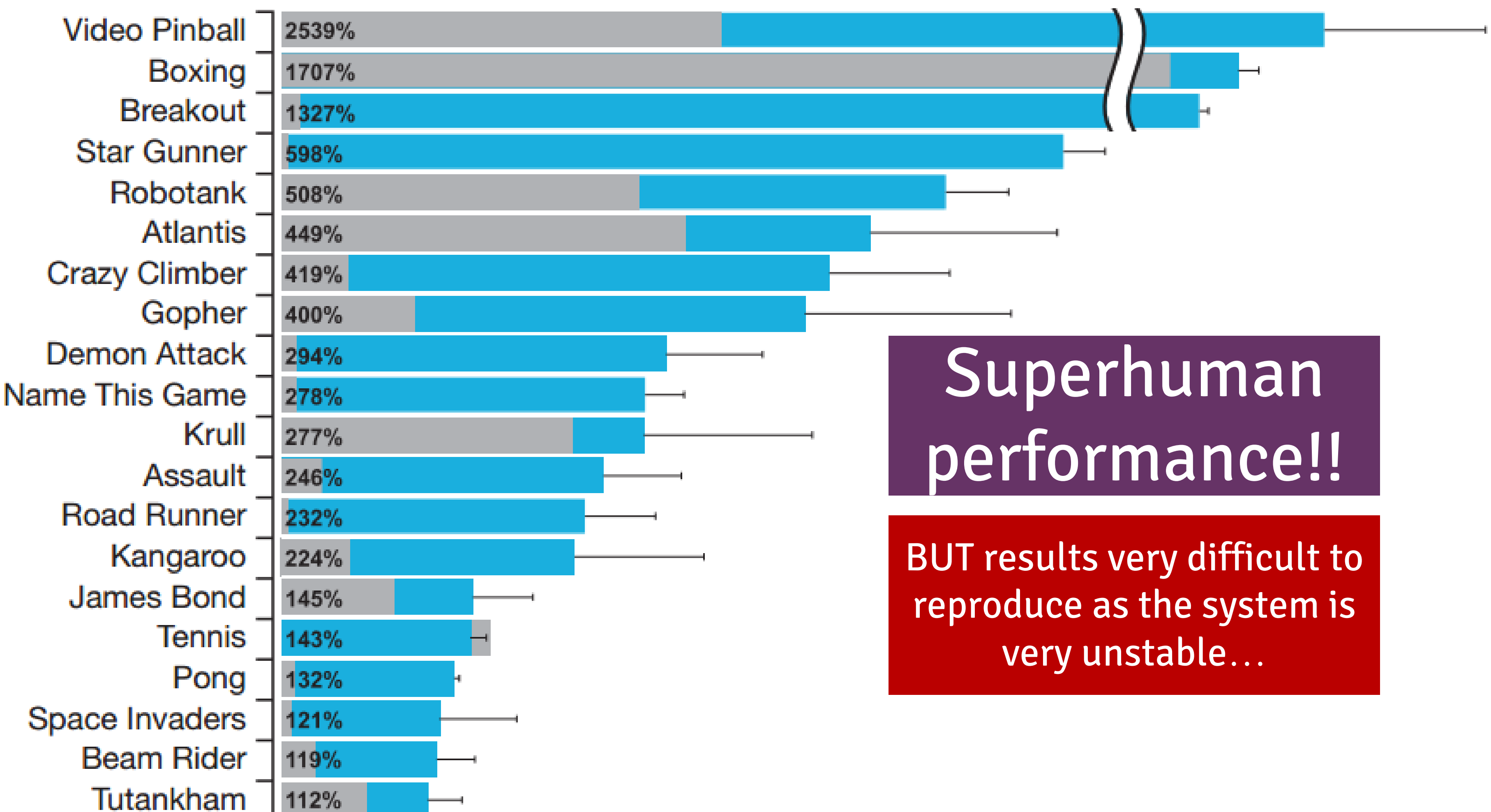
Minimize the loss

$$\mathbf{E}_{(X,A,R,X')\sim D}\left[\left(R+\gamma\max_b Q_{\theta_k}(X',b)-Q_\theta(X,A)\right)^2\right]$$

+ training tricks:

- Store transitions (x,a,r,x') in replay buffer D to break dependence on recent samples
- Compute small updates by mini-batch stochastic gradient descent
- Use an older parameter vector θ_{k-m} in target to avoid oscillations
- …

DEEP Q NETWORKS FOR PLAYING ATARI



Superhuman performance!!

BUT results very difficult to reproduce as the system is very unstable…

THIS SHORT COURSE: A PRIMAL-DUAL VIEW

part 1

- Markov decision processes
  - Value functions and optimal policies
- Primal view: Dynamic programming
  - Policy evaluation, value and policy iteration
  - Value-function-based methods
    - Temporal differences, Q-learning, LSTD, deep Q networks,…

part 2

- Dual view: Linear programming
  - LP duality in MDPs
  - Direct policy optimization methods
    - Policy gradients, REPS, TRPO,…

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A PRIMAL-DUAL VIEW

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- Markov decision processes
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  - Value-fu…
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But first:
some more notation ☺

part 2

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POLICIES AND TRAJECTORY DISTRIBUTIONS

Policy: mapping from histories to actions

$$\pi: x_1, a_1, x_2, a_2, \dots, x_t \mapsto a_t$$

Stationary policy: mapping from states to actions
(no dependence on history or t)

$$\pi: x \mapsto a$$

Let $\tau = (x_1, a_1, x_2, a_2, \dots)$ be a trajectory generated by running π in the MDP $\tau \sim (\pi, P)$:

- $a_t = \pi(x_t, a_{t-1}, x_{t-1}, \dots, x_1)$
- $x_{t+1} \sim P(\cdot\,|x_t, a_t)$

Expectation under this distribution: $\mathbf{E}_\pi[\cdot]$

POLICIES AND TRAJECTORY DISTRIBUTIONS

Stationary stochastic policy: mapping from states to action distributions

$$\pi: A \times X \to [0,1]$$

where

$$\pi(a|x) = P[a_t = a | x_t = x]$$

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ANOTHER PERSPECTIVE ON DISCOUNTED REWARDS

Discounted MDPs:

- No terminal state
- Discount factor $\gamma \in (0,1)$
- GOAL: maximize total discounted reward

$$R_\gamma = \mathbf{E}[\sum_{t=0}^{\infty} \gamma^t r_t]$$

ANOTHER PERSPECTIVE ON DISCOUNTED REWARDS

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&= \sum_{x,a} \mu_\pi(x,a) r(x,a)
\end{aligned}$$

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Observe: the discounted reward of a policy is

$$R_\gamma^\pi = \langle \mu_\pi, r \rangle$$

μ_π = the discounted occupancy measure induced by policy π:

$$\mu_\pi(x,a) = \sum_{t=0}^{\infty} \gamma^t \mathbf{P}_\pi[x_t = x, a_t = a]$$

ANOTHER PERSPECTIVE ON DISCOUNTED REWARDS

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A linear optimization problem?!

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TOWARDS A LINEAR-PROGRAM FORMULATION

Theorem

A function μ is a discounted occupancy measure of some (stationary stochastic) policy π if and only if it satisfies

$$\sum_{a'} \mu(x', a') = (1 - \gamma) \sum_{a'} \mu_0(x', a') + \gamma \sum_{x,a} P(x'|x, a)\mu(x, a)$$

and $\sum_{x,a} \mu(x, a) = 1/(1 - \gamma)$.

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Linear constraints!

Define $\Delta =$ the set of occupancy measures μ.

OPTIMIZATION IN MDPS AS A LINEAR PROGRAM

LP

$$R_\gamma^* = \max_{\mu \in \Delta} \langle \mu, r \rangle$$

OPTIMIZATION IN MDPS AS A LINEAR PROGRAM

LP

$$R_\gamma^* = \max_{\mu \in \Delta} \langle \mu, r \rangle$$

LP'

$$R_\gamma^* = \min_{V \in \mathbb{R}^X} \langle \mu_0, V \rangle$$

s.t. $V(x) \geq r(x,a) + \gamma \sum_y P(y|x,a) V(y) \quad (\forall x, a)$

OPTIMIZATION IN MDPS AS A LINEAR PROGRAM

Dual LP

$$R_\gamma^* = \max_{\mu \in \Delta} \langle \mu, r \rangle$$

Primal LP

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*names are due to tradition

OPTIMIZATION IN MDPS AS A LINEAR PROGRAM

Dual LP

$$R_\gamma^* = \max_{\mu \in \Delta} \langle \mu, r \rangle$$

Primal LP ≡ The Bellman opt. equations

$$V^*(x) = \max_a \left\{ r(x,a) + \gamma \sum_y P(y|x,a) V^*(y) \right\}$$

Assuming $\mu_0 > 0$

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OPTIMIZATION IN MDPS AS A LINEAR PROGRAM

A single numerical objective to optimize!

Dual LP

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Primal LP $\equiv$ The Bellman opt. equations

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Theorem

There exists a basic solution $\mu^* \in \Delta$ to the dual LP.

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"Proof":

objective $\langle \mu, r \rangle$ is bounded on nonempty Δ

$$\Rightarrow$$

there exists optimal solution $\mu^* \in \Delta$

$$\Rightarrow$$

there exists basic solution $\mu^* \in \Delta$

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A "corner" of Δ

there exists basic solution $\mu^* \in \Delta$

EXTRACTING A POLICY

? **Question:** how do we extract a policy from a feasible $\mu \in \Delta$?

EXTRACTING A POLICY

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Corollary

Assume that $\mu_0(x) > 0$ for all $x \in X$. Then, for any occupancy measure $\mu \in \Delta$, there exists a unique policy π such that $\mu = \mu_\pi$, given by

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Basic solutions
$\Leftrightarrow$
Deterministic policies

Well-defined since
$\sum_b \mu(x,b) > 0$ by assumption

LINEAR PROGRAMMING FOR MDPS

"Why don't they teach this in school?!?"

- Needs some strange conditions that DP theory does not ($\mu_0 > 0$ for existence results and for optimal policy)
- Temporal aspect is rather abstract
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THIS SHORT COURSE:
A PRIMAL-DUAL VIEW

part 1

- Markov decision processes
  - Value functions and optimal policies
- Primal view: Dynamic programming
  - Policy evaluation, value and policy iteration
  - Value-function-based methods
    - Temporal differences, Q-learning, LSTD, deep Q networks,…

part 2

- Dual view: Linear programming
  - LP duality in MDPs
  - Direct policy optimization methods
    - Policy gradients, REPS, TRPO,…

DIRECT POLICY OPTIMIZATION

Idea: derive algorithms by thinking of $\mu \in \Delta$ as the decision variable!

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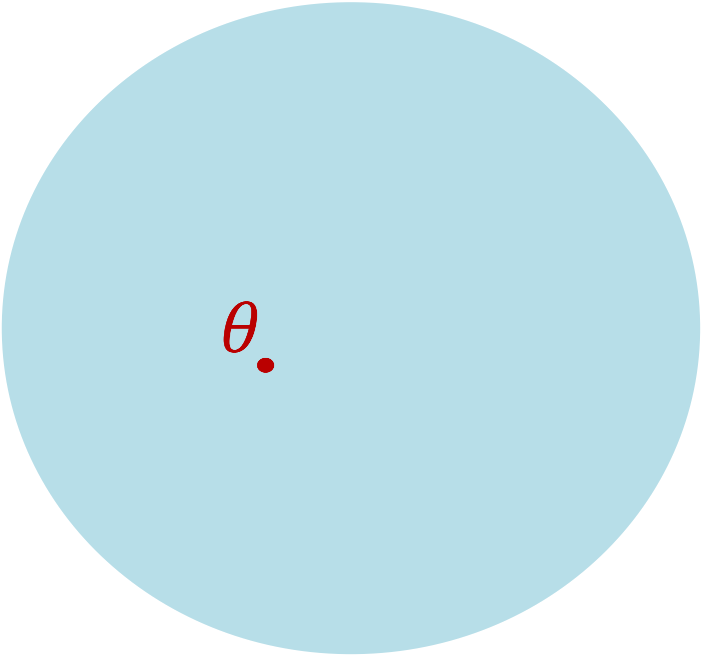
POLICY GRADIENT METHODS

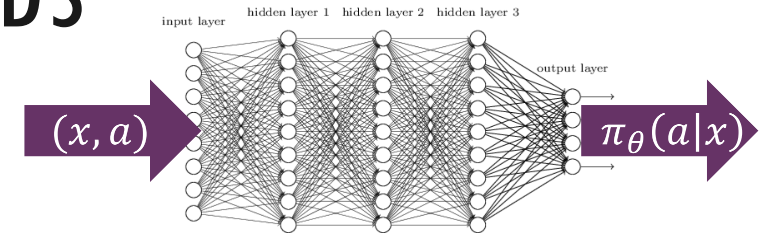
Parameter space Θ



- Construct mapping

$$\theta \mapsto \pi_\theta$$

POLICY GRADIENT METHODS



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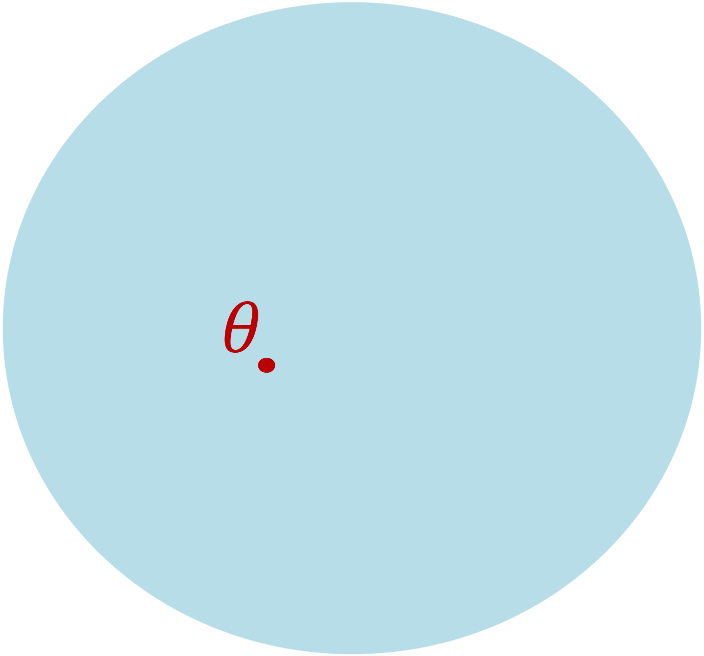

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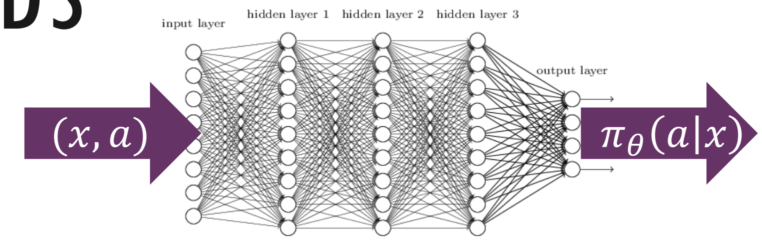

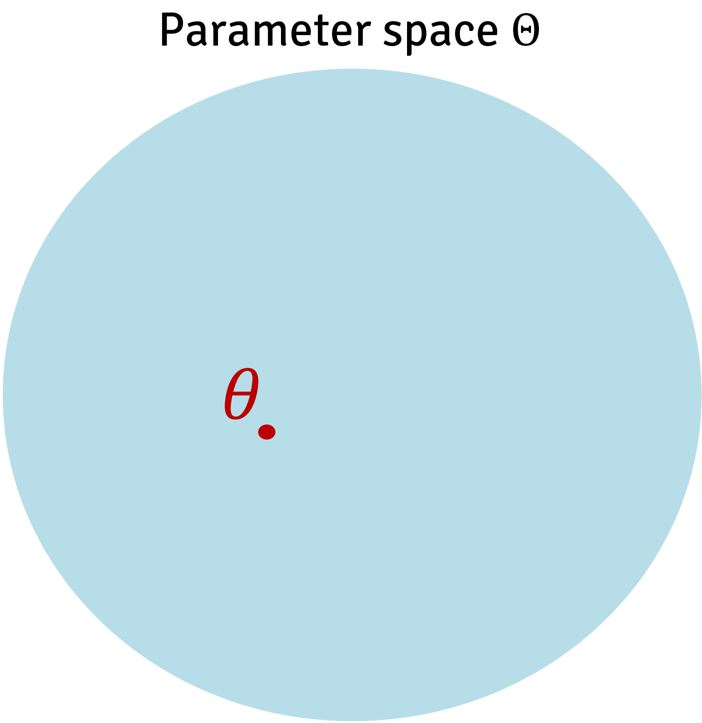

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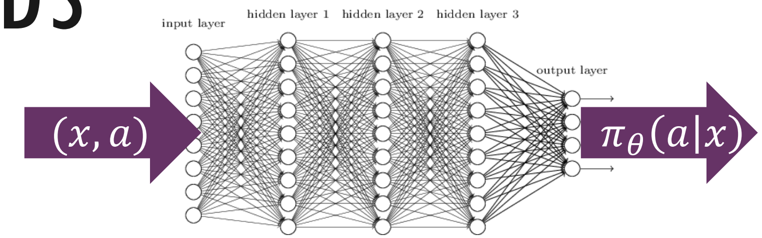

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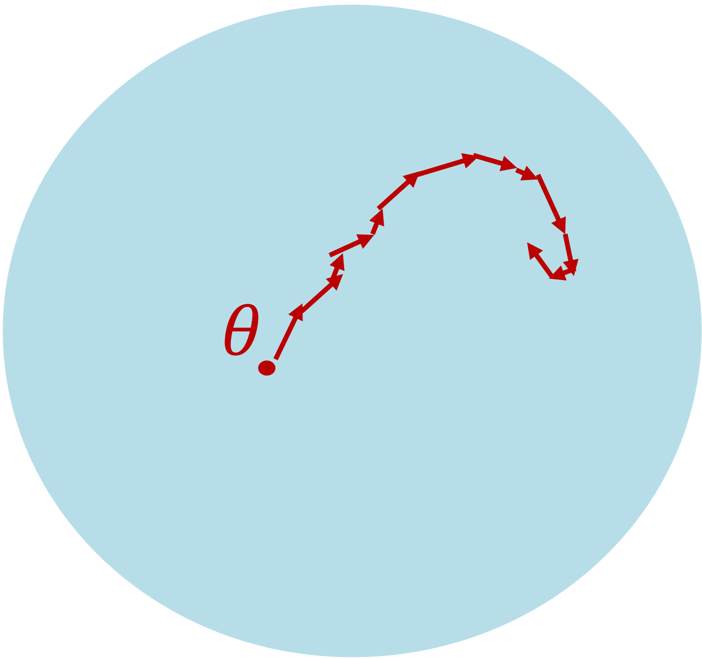

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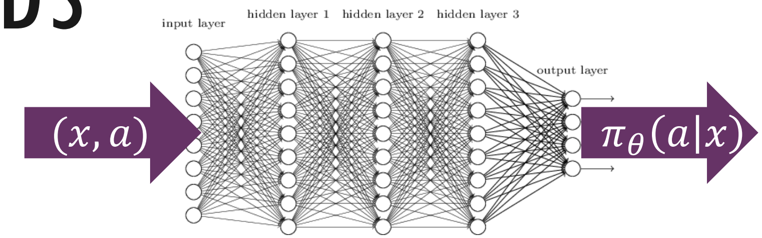

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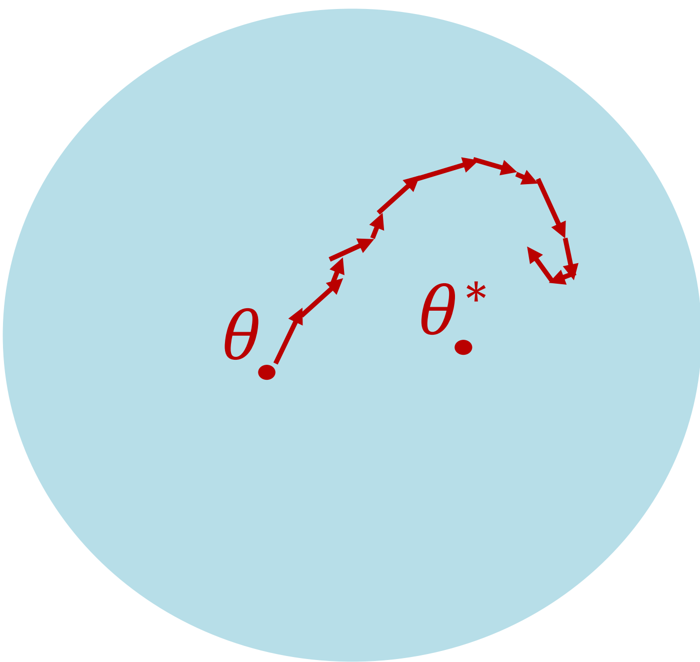

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… and hope for convergence

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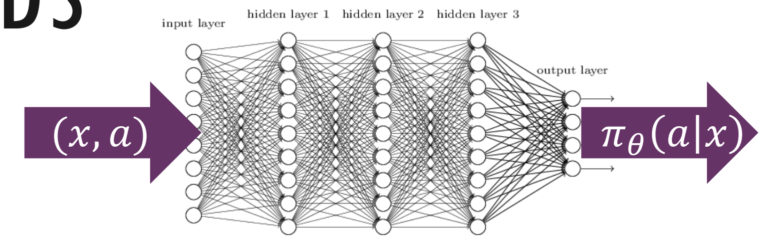

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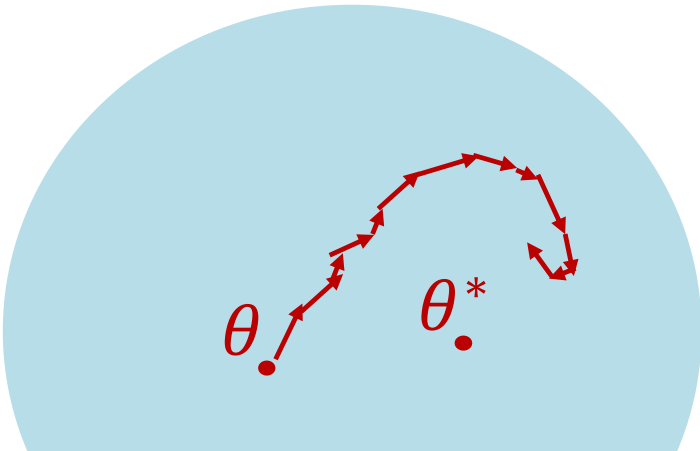

How can we estimate the gradients?

- Construct mapping
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THE POLICY GRADIENT THEOREM

Theorem

$$\nabla_\theta \rho(\theta) = \sum_x \mu_\theta(x) \sum_a \nabla_\theta \pi_\theta(a|x) Q^{\pi_\theta}(x,a)$$

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Gradient can be written as an expectation!!!!

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REINFORCE: A STOCHASTIC POLICY GRADIENT ALGORITHM

Idea: replace expectation by a sample mean ⇒ stochastic gradient algorithm

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REINFORCE

Input: arbitrary initial θ_0

For $k = 0,1, \ldots$

- Obtain sample trajectory $(x_t, a_t, r_t)_{t=1}^{T} \sim \pi_{\theta_k}$
- Estimate $\hat{Q}_k \approx Q^{\pi_{\theta_k}}$ by Monte Carlo
- Estimate $g_k \approx \nabla_\theta \rho(\theta_k)$ by the average of

$$g_{k,t} = \nabla_\theta \log \pi_{\theta_k}(a_t|x_t)\hat{Q}_k(x_t, a_t)$$

- Update $\theta_{k+1} = \theta_k + \alpha_k g_k$

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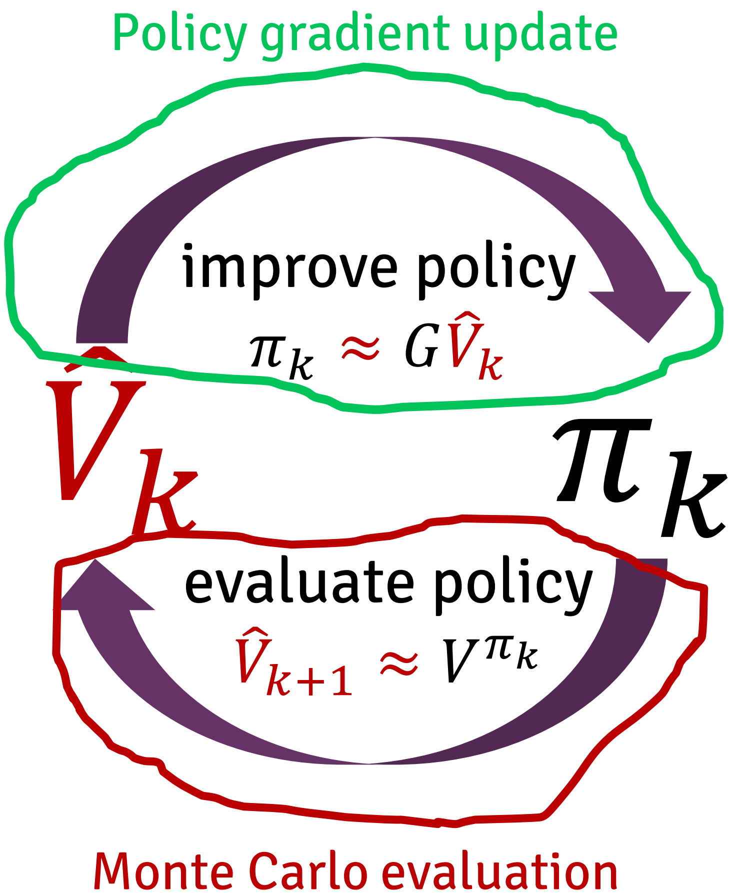
$$\mathbf{E}[g_k] = \nabla_\theta \rho(\theta_k)$$

REINFORCE AS DIRECT POLICY SEARCH



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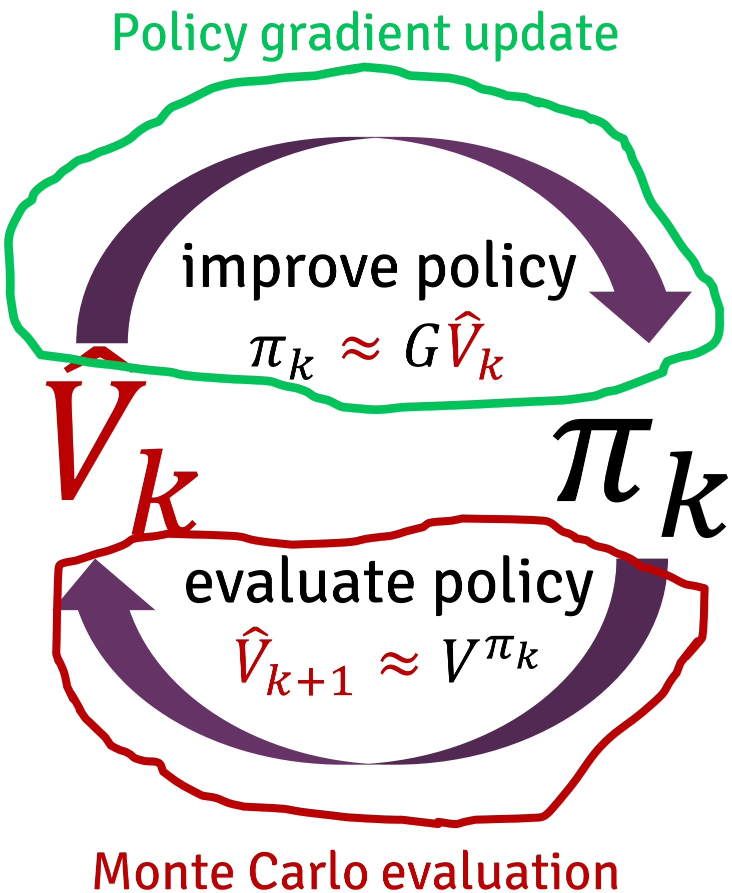


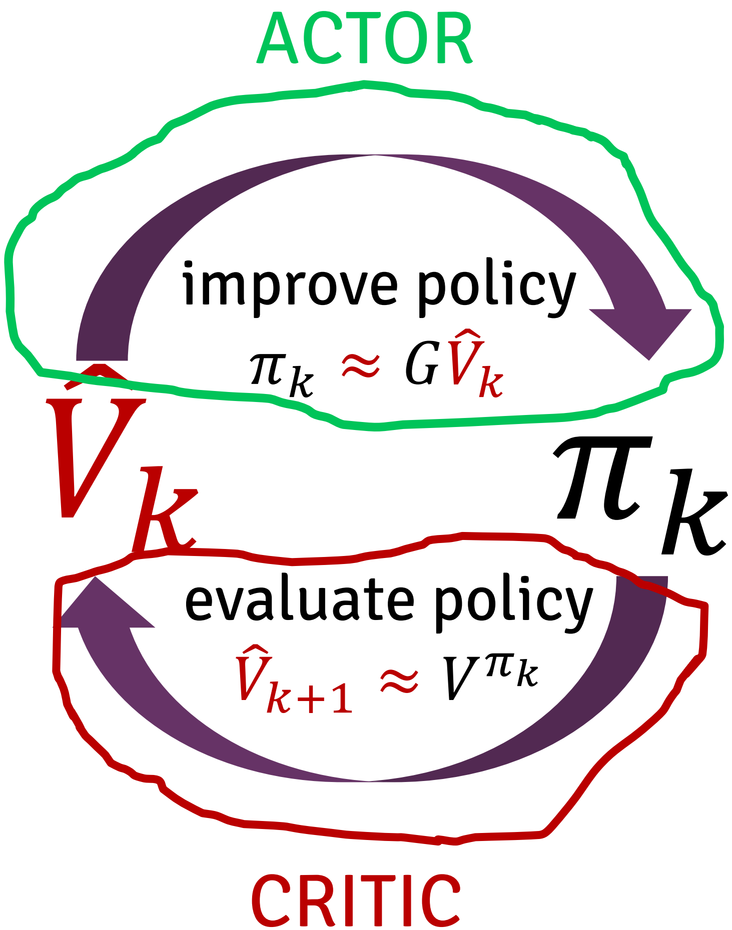
☺ direct method: no explicit approximation of V^{π} ☺

☺ converges to local optimum ☺

☺ less aggressive updates ☺

☹ large variance of g_k ☹

ACTOR-CRITIC METHODS



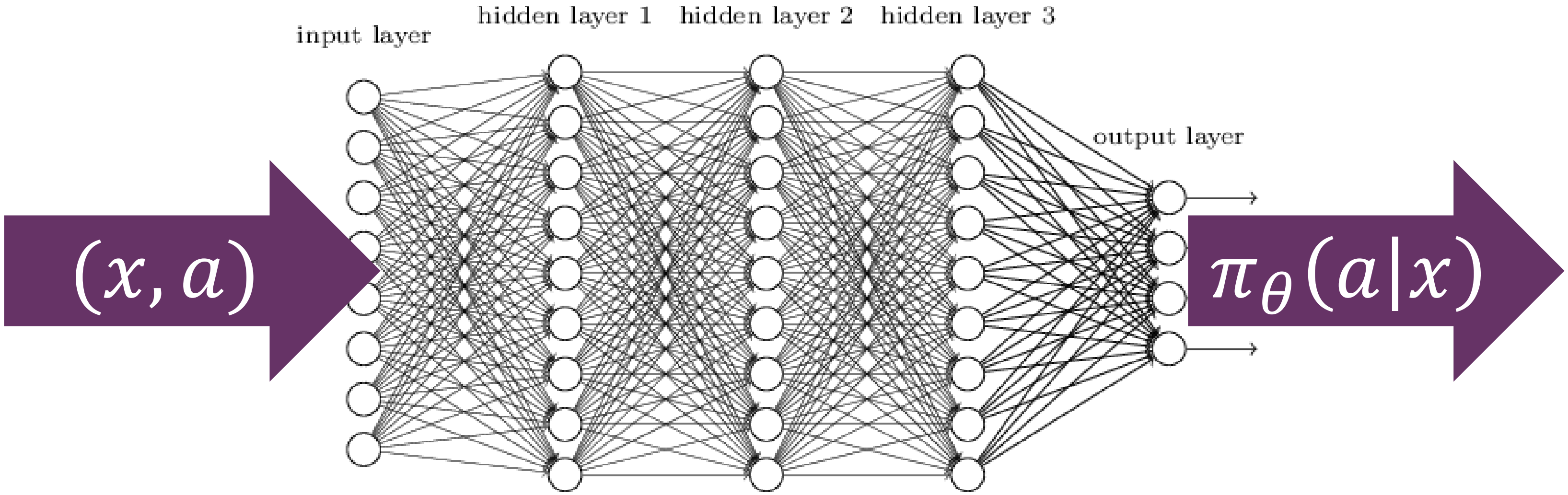
Typical actor:
policy gradient updates

Critic:

- Monte Carlo ⇒ REINFORCE
- TD(λ)
- LSTD(λ)
- DQN, …

A TYPICAL DEEP RL ARCHITECTURE: A3C

Parametrize policy by a deep neural net



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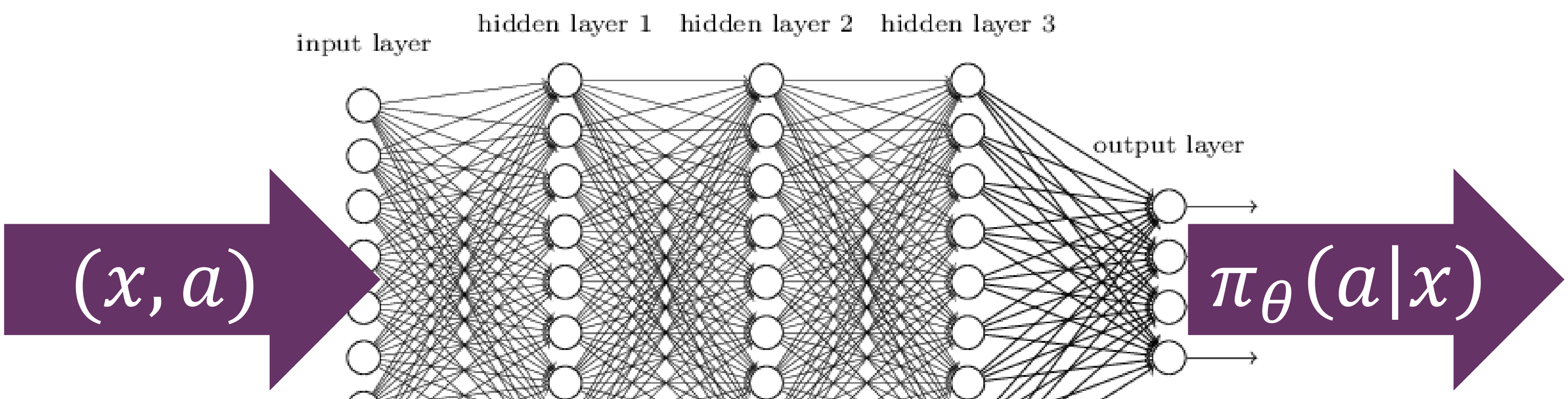

+ another neural net to estimate V^{π_θ} and to estimate Q^{π_θ} by "bootstrapped" Monte Carlo
+ asynchronous updates
+ entropy-regularization of the objective
+…

POLICY GRADIENTS: THE FINAL ANSWER?

Policy gradient update

$$\theta_{t+1} = \arg\max_{\theta} \left\{ \langle \theta, \nabla \rho(\theta_t) \rangle - \frac{1}{\alpha_t} \|\theta - \theta_t\|_2^2 \right\}$$

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Euclidean norm may be unnatural way to measure distance between μ_θ and μ_{θ_t}?

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Linearizing ρ at θ_t may lead to instability?

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+ Issue #3:
Policy gradient estimator has huge variance ☹

A BETTER APPROACH: SMOOTHED LINEAR PROGRAMS

Dual LP

$$R_\gamma^* = \max_{\mu \in \Delta} \langle \mu, r \rangle$$

A BETTER APPROACH: SMOOTHED LINEAR PROGRAMS

Dual convex program

$$\tilde{R}_{\gamma}^{*} = \max_{\mu \in \Delta} \left\{ \langle \mu, r \rangle + \frac{1}{\eta} \Phi(\mu) \right\}$$

A BETTER APPROACH: SMOOTHED LINEAR PROGRAMS

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$$\tilde{R}_\gamma^* = \max_{\mu \in \Delta} \left\{ \langle \mu, r \rangle + \frac{1}{\eta} \Phi(\mu) \right\}$$

Φ: strongly convex function of μ:

- smooth optimum

$$\mu^* = \arg\max_{\mu} \left\{ \langle \mu, r \rangle + \frac{1}{\eta} \Phi(\mu) \right\} = \frac{1}{\eta} \nabla_r \Phi^*(\eta r)$$

- regularization effect ⇒ better generalization?

BETTER PROXIMAL REGULARIZATION: MIRROR DESCENT

Policy gradient update

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$$\mu_{t+1} = \arg\max_{\mu \in \Delta} \left\{ \langle \mu, r \rangle - \frac{1}{\eta_t} D(\mu | \mu_t) \right\}$$

BETTER PROXIMAL REGULARIZATION: MIRROR DESCENT

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$$\ldots \theta_t)\rangle - \frac{1}{\alpha_t}\|\theta - \theta_t\|_2^2 \Big\}$$

No need for local linearization

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Proximal regularization through Bregman divergence $D(\mu|\mu')$
(strongly convex in μ)

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Closed-form "policy update":

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"Value function"

$\tilde{V}_t = ???$

THE REPS VALUE FUNCTION

Theorem

The REPS value function $\tilde{V}_t$ is given as the minimizer of the loss function

$$\tilde{L}(V) = \log \mathbf{E}_{x\sim\mu_t}\left[e^{\eta_t\left(T^\pi V(x) - V(x)\right)}\right]$$

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"Proof": Lagrangian duality.

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"Proof": Lagrangian duality.

A natural competitor for the Bellman error

$$L(V) = \mathbf{E}_{x\sim\mu}\left[\left(T^\pi V(x) - V(x)\right)^2\right]???$$

Stay tuned for "deep REPS" results ☺

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THE REGULARIZED BELLMAN EQUATIONS

The Bellman opt. equations

$$V^*(x) = \max_a \left\{ r(x,a) + \gamma \sum_y P(y|x,a) V^*(y) \right\}$$

THE REGULARIZED BELLMAN EQUATIONS

The **regularized** Bellman opt. equations

$$V^*(x) = \underset{a}{\text{softmax}}^{\eta}\left\{r(x,a) + \gamma \sum_y P(y|x,a)V^*(y)\right\}$$

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Used almost exclusively since ~late 2016

- Better optimization properties: smooth gradients, less sensitive to errors
- Better exploration: optimal policy naturally stochastic, no need for ε −greedy trick

THE REGULARIZED BELLMAN EQUATIONS

Is there a natural "dual" explanation?

The regularized Bellman opt. equations

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DUALITY THEORY FOR THE REGULARIZED BELLMAN EQUATIONS

The regularized Bellman opt. equations

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??? Dual convex program ???

$$\tilde{R}_\gamma^* = \max_{\mu \in \Delta} \left\{ \langle \mu, r \rangle - \frac{1}{\eta} \Phi(\mu) \right\}$$

DUALITY THEORY FOR THE REGULARIZED BELLMAN EQUATIONS

Theorem (Neu et al., 2017)
The two formulations are connected by Lagrangian duality with the choice

$$\Phi(\mu) = \sum_{x,a} \mu(x,a) \log \frac{\mu(x,a)}{\sum_b \mu(x,b)}$$

$$= \sum_{x} \mu(x) \sum_{a} \pi_\mu(a|x) \log \pi_\mu(a|x)$$

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The conditional entropy of $A|X$ under μ

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The conditional entropy of $A|X$ under μ

A convex function of μ!

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Dual convex program

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MIRROR DESCENT WITH CONDITIONAL ENTROPY (NEU ET AL., 2017)

Mirror descent update

$$\mu_{t+1} = \arg\max_{\mu \in \Delta} \left\{ \langle \mu, r \rangle - \frac{1}{\eta_t} D_\Phi(\mu | \mu_t) \right\}$$

$$D_\Phi(\mu | \mu_t) = \sum_{x,a} \mu(x, a) \log \frac{\pi_\mu(a|x)}{\pi_t(x,a)}$$

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Closed-form policy update:

$$\pi_{t+1}(a|x) = \pi_t(a|x) e^{\eta_t \left(r(x,a) + \gamma \mathbf{E}_{y|x,a}[\widetilde{V}_t(y)] - \widetilde{V}_t(x) \right)}$$

MIRROR DESCENT WITH CONDITIONAL ENTROPY (NEU ET AL., 2017)

Mirror descent update

$$\mu_{t+1} = \arg\max_{\mu \in \Delta} \left\{ \langle \mu, r \rangle - \frac{1}{\eta_t} D_\Phi(\mu | \mu_t) \right\}$$

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Value function $\tilde{V}_t$ = solution to proximally regularized BOE

TRUST-REGION POLICY OPTIMIZATION (TRPO, SCHULMAN ET AL., 2015)

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TRUST-REGION POLICY OPTIMIZATION (TRPO, SCHULMAN ET AL., 2015)

Dense surrogate for $\langle \mu, r \rangle$

(works because $\langle \mu, r \rangle = \langle \mu, \tilde{Q}_t - \tilde{V}_t \rangle$ when $\mu \in \Delta$)

Mirror descent update

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$$D_\Phi(\mu|\mu_t) = \sum_x \mu_t(x) \sum_a \pi_\mu(a|x) \log \frac{\pi_\mu(a|x)}{\pi_t(x,a)}$$

$\mu_t \approx \mu_{t+1}$, but μ_t can be sampled from

TRUST-REGION POLICY OPTIMIZATION (TRPO, SCHULMAN ET AL., 2015)

Theorem (Neu et al., 2017)
TRPO is equivalent to the MDP-E algorithm of Even-Dar, Kakade and Mansour (2006)

$$\Rightarrow$$

$$\lim_{t \to \infty} \langle \mu_t, r \rangle = \langle \mu^*, r \rangle$$

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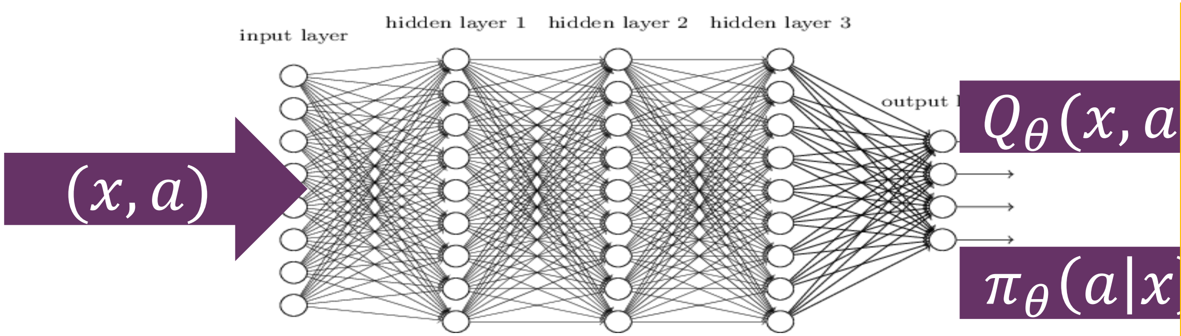

+ more tricks:
- Another surrogate for μ
- Truncation of objective
- Constraint vs. penalty
- Mini-batch SGD
- …

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Literally the most broadly used deep RL algorithm!
(but reading the original paper is not recommended…)

+ more tricks:

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BEYOND LINEAR PROGRAMMING: SADDLE-POINT OPTIMIZATION

Dual LP

$$R_\gamma^* = \max_{\mu \in \Delta} \langle \mu, r \rangle$$

Primal LP

$$R_\gamma^* = \min_{V \in \mathbb{R}^X} \langle \mu_0, V \rangle$$

s.t. $V(x) \geq r(x,a) + \gamma \sum_y P(y|x,a) V(y) \quad (\forall x, a)$

BEYOND LINEAR PROGRAMMING: SADDLE-POINT OPTIMIZATION

Bellman saddle point

$$\min_{V} \max_{\mu \in \Delta} \{ \langle \mu, r + \gamma P V - V \rangle + (1-\gamma) \langle \mu_0, V \rangle \}$$

BEYOND LINEAR PROGRAMMING: SADDLE-POINT OPTIMIZATION

Bellman saddle point

$$\min_V \max_{\mu\in\Delta}\{\langle\mu, r+\gamma PV - V\rangle + (1-\gamma)\langle\mu_0, V\rangle\}$$

≈ the Lagrangian of the two LPs

$\Rightarrow$

solution exists & optimal policy can be extracted under same conditions

PRIMAL-DUAL π-LEARNING (WANG ET AL., 2017-)

Bellman saddle point

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Value update:

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Policy update:

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Exponentiated gradient step in dual

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$\mu_t P)$

Gradient step in primal

≈ incremental REPS

state-of-the art sample complexity results for discounted & undiscounted MDPs!

Exponentiated gradient step in dual

$\mu_{t+1}(x,a) = \mu_t(x,a)e^{\eta(\ldots_{y|x,a}[\widetilde{V}_t(y)] - \widetilde{V}_t(x))}$

THIS SHORT COURSE: A PRIMAL-DUAL VIEW

part 1

- Markov decision processes
  - Value functions and optimal policies
- Primal view: Dynamic programming
  - Policy evaluation, value and policy iteration
  - Value-function-based methods
    - Temporal differences, Q-learning, LSTD, deep Q networks,…

part 2

- Dual view: Linear programming
  - LP duality in MDPs
  - Direct policy optimization methods
    - Policy gradients, REPS, TRPO,…

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what else?

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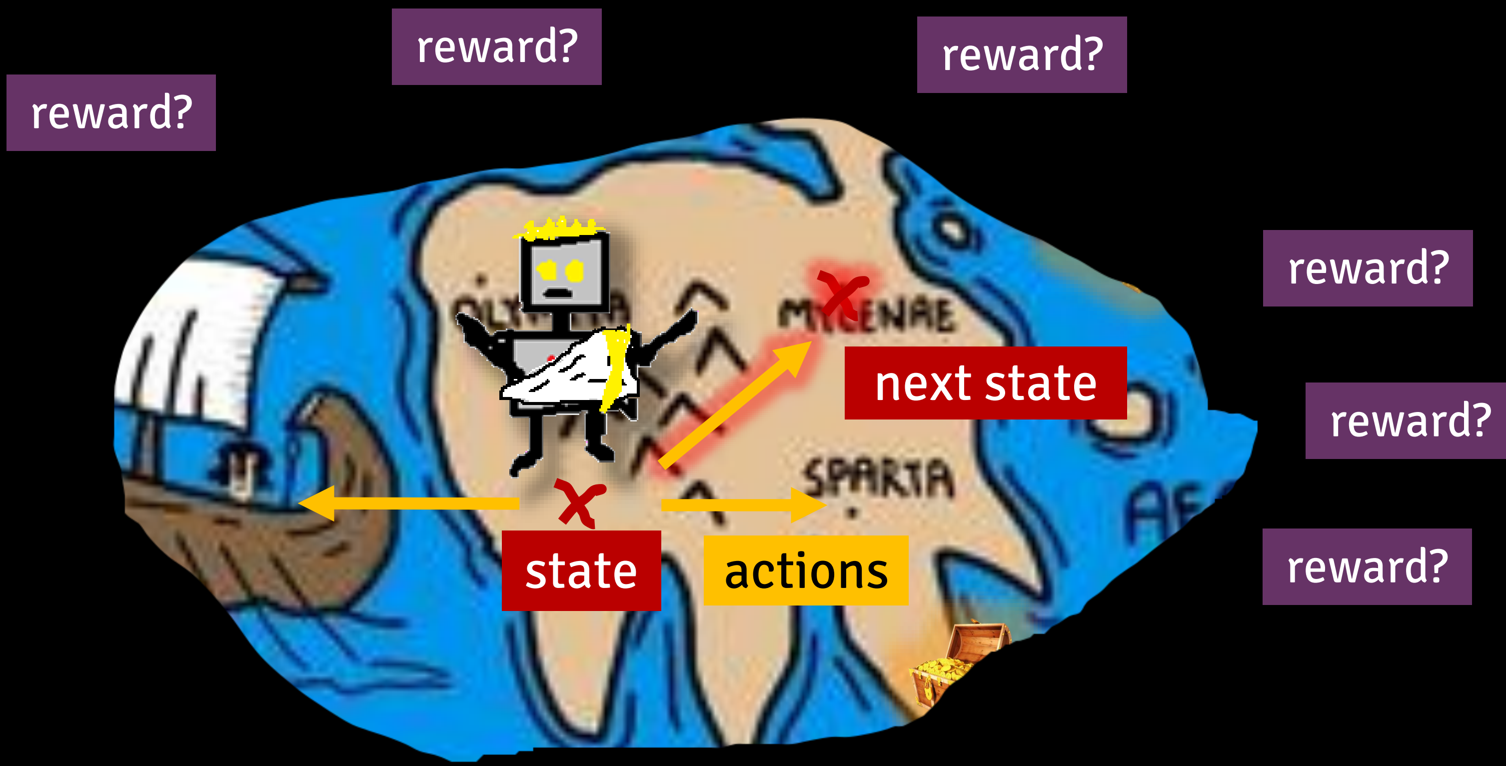
EXPLORATION VS. EXPLOITATION



EXPLORATION VS. EXPLOITATION

reward?

reward?

reward?

reward?

next state

reward?

reward?

- Multi-armed bandits
- Exploration bonuses
- Thompson sampling
- Monte Carlo tree search
- …

EXPLORATION VS. EXPLOITATION

Still no practical algorithms!

- Multi-armed bandits
- Exploration bonuses
- Thompson sampling
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- …

CONCLUSION

RL is an insanely popular field with

- huge recent successes
- some beautiful fundamental theory
- unique algorithmic ideas

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BUT still fundamental challenges in

- understanding efficient exploration
- stability of algorithms
- generalizability of successes

CONCLUSION

Come and work on RL theory ;)

BUT still fundamental challenges in

- understanding efficient exploration
- stability of algorithms
- generalizability of successes

CONCLUSION

Come and work on RL theory ;)

+ also come see
PARADISE LOST
tonight!

Thanks!!!