Modeling exchange rates volatility through flexible threshold models

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Outline

1. The problem of interest
2. A flexible Threshold model
3. A Population-based RJ algorithm
4. Incorporating GARCH effects
5. A comparative Spline-GARCH model
6. Empirical results
7. Extension to the Multivariate case

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The Problem

Model:

$$y_t = X_t\beta + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_t^2)$$
$$\sigma_t^2 = f(Z_{i,\lambda_t}),$$

where Z_i, $i = 1, ..., K$ is a set of exogenous variables (i.e. news) affecting the variance process in a nonlinear way. These variables may be observed on lower frequencies than y_t and λ_t denotes the last observation of Z_i before time t.

- Target:
  a) Estimate (and predict) the variance σ_t^2, through a flexible threshold model.
  b) Perform model selection and identify the macroeconomic news announcements that mostly affect the variance process of exchange rates.
- Why threshold model: As opposed to smooth transition models (i.e. splines, wavelets, neural nets etc.) threshold models allow for sudden jumps in the volatility, a desired property when there are news announcements.

Review (1)

(SE)TAR (Tong (1983); Chan and Tong (1986))

$$y_t = \begin{cases} \beta_0^{(1)} + \sum_{i=1}^{p} \beta_i^{(1)} y_{t-i} + \sigma^{(1)} \epsilon_t & \text{if } y_{t-d} < c \\ \beta_0^{(2)} + \sum_{i=1}^{p} \beta_i^{(2)} y_{t-i} + \sigma^{(2)} \epsilon_t & \text{if } y_{t-d} \geq c \end{cases}$$

General form: $y_t = \beta_0^{(j)} + \sum_{i=1}^{p} \beta_i^{(j)} y_{t-i} + \sigma^{(j)} \epsilon_t \quad \text{if } c_{j-1} \leq y_{t-d} < c_j$,

where c_j are the thresholds points for $j = 1, \ldots, J$ regimes.

SETAR-TWO (Pfann et al. (1996))

$$y_t = \left\{ \begin{array}{ll} \beta_0^{(1)} + \sum_{i=1}^{p} \beta_i^{(1)} y_{t-i} & \text{if } y_{t-d} < c_1 \\ \beta_0^{(2)} + \sum_{i=1}^{p} \beta_i^{(2)} y_{t-i} & \text{if } y_{t-d} \geq c_1 \end{array} \right\} + \left\{ \begin{array}{ll} \sigma^{(1)} \epsilon_t & \text{if } y_{t-d} < c_2 \\ \sigma^{(2)} \epsilon_t & \text{if } y_{t-d} \geq c_2 \end{array} \right\}$$

Difference among TAR and SETAR models: In SETAR models the threshold points c_j are related to the dependent variable y, while in TAR models with an exogenous variable.

Review (2)

DTGARCH (Brooks (2001); Chen et al. (2003))

$$\text{if } c_{j-1} \leq y_{t-d} < c_j \text{ then:}$$
$$y_t = \beta_0^{(j)} + \sum_{i=1}^{p} \beta_i^{(j)} y_{t-i} + \epsilon_t, \quad \epsilon_t \sim N(0, h_t)$$
$$h_t = \alpha_0^{(j)} + \alpha_1^{(j)} \epsilon_{t-1}^2 + \alpha_2^{(j)} h_{t-1}.$$

Flexible Threshold model (Dellaportas et al. (2007))

$$y_t = \beta_0 + \beta_1 y_{t-1} + \sigma_t(\Phi_{t-1})\epsilon_t$$
$$\sigma_t^2(\Phi_{t-1}) = \sigma^2 \left(1 + \sum_{j=1}^{J} \gamma_j F_j(\Phi_{t-1})\right)$$
$$F_j(\Phi_{t-1}) = \left[s_j(z_{j,t-1} - c_j)\right],$$

where γ_j is the size of jump associated with the jth threshold function $F_j(\Phi_{t-1})$, which depends on the information set available at the previous time point Φ_{t-1}. The constant parameter σ^2 is thought as the global static variance that remains unchanged through time. The threshold function $F_j(\Phi_{t-1})$ takes value 1 if $\left[s_j(z_{j,t-1} - c_j)\right] > 0$ and 0 otherwise, z_j is an explanatory variable on which F_j splits, c_j is the threshold point, and $s_j \in \{-1, 1\}$ is required so that F_j can be nonzero either when $z_{j,t-1} > c_j$(with $s_j = 1$) or when $z_{j,t-1} \leq c_j$(with $s_j = -1$). This setup is combined with model selection on the number as well as the position of the thresholds.

The dataset

- y: 849 daily observations, from 1/1/2002 up to 1/4/2005 (weekends excluded), of the euro-dollar log-returns. From these we keep 784 observations (3 years) as a fitted sample and the rest 65 (3 months) are used for the out of sample forecasts.
- Z_i: 15 U.S. scheduled monthly macroeconomic announcements. Their values represent the absolute percentage difference of the outcome from their consensus (as this is reported in Bloomberg).

List of macroeconomic announcements

	Announcement	Distinct values	Mean	St. dev.	Median	Maximum
1	Advance Retail Sales	12	0.0038	0.0038	0.0030	0.0150
2	Consumer Confidence	36	0.0449	0.0436	0.0285	0.1688
3	Consumer Price Index	4	0.0009	0.0009	0.0010	0.0030
4	Durable Goods Orders	26	0.0170	0.0154	0.0115	0.0720
5	GDP Annualized	13	0.0043	0.0037	0.0035	0.0130
6	Housing Starts	36	0.0477	0.0297	0.0427	0.1238
7	Industrial Production	6	0.0023	0.0015	0.0020	0.0050
8	ISM Manufacturing	36	0.0275	0.0261	0.0181	0.0940
9	ISM Non-Manufacturing	36	0.0494	0.0330	0.0464	0.1510
10	Leading Indicators	4	0.0010	0.0010	0.0010	0.0040
11	Personal Income	5	0.0011	0.0011	0.0010	0.0040
12	Producer Price Index	10	0.0038	0.0034	0.0030	0.0120
13	Trade Balance	35	0.0546	0.0414	0.0492	0.1872
14	Unemployment rate	4	0.0011	0.0010	0.0010	0.0030
15	Wholesale Inventories	10	0.0036	0.0024	0.0030	0.0090

Notes: The diagnostics refer to the fitted sample (3 years). All types of announcements are monthly, thus they all have 36 observations in the fitted sample. However the number of observations that are distinct differs by type of announcement. (column: distinct values).

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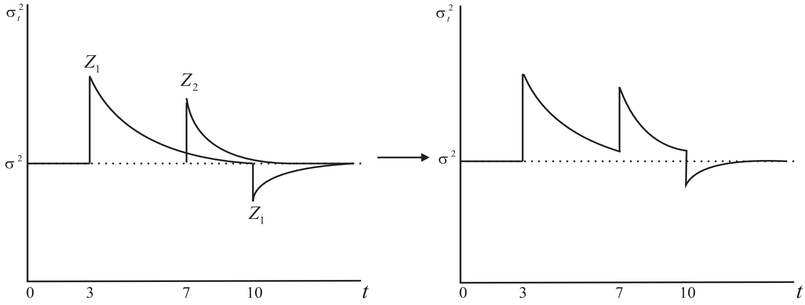
The Threshold Model

$$y_t = X_t\beta + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \sigma^2 \left[1 + \sum_{i=1}^{K} \sum_{j=1}^{J_i} \left([\gamma_{i,j} \exp(-r_{i,j}(t - \lambda_{i,t}))] I_{i,j} \right) + \sum_{i=1}^{K} (s_i S_{i,q}) \right]$$

- $I_{i,j}$: an indicator variable taking value 1 if $c_{i,j} \leq Z_{i,\lambda_{i,t}} < c_{i,j+1}$ and zero otherwise.
- Z_i: a set of exogenous variables $i = 1, 2, ..., K$, maybe observed on lower frequencies than y, affecting the variance process under $j = 1, 2, ..., J_i$ possible regimes.
- $\lambda_{i,t}$: the time of the last observation of Z_i until time t.
- $c_{i,j}$: the threshold points associated with variable Z_i under the jth regime.
- $S_{i,q}$: an indicator variable taking value 1 at the interval $\{t - q, ..., t - 1\}$ if the variable Z_i is observed at time t and zero otherwise.
- β, σ^2, γ, s and r are coefficients of appropriate order to be estimated.



At the initial stage, σ_t^2 equals the global static variance σ^2. At $t = 3$ there is an observation of variable Z_1, which causes a certain jump in the volatility (of size $\gamma_{1,1}$) that diminishes on a certain rate ($r_{1,1}$). At $t = 7$ there is the observation of another variable Z_2 that causes another jump (of size $\gamma_{2,1}$). As shown in the right panel, the effect of the two jumps is additive. Finally at $t = 10$ we have a new observation of variable Z_1 causing at this case a negative jump (of size $\gamma_{1,2}$) in the volatility. The size and sign of a jump depends on the regime, that is on whether the value of the exogenous variable is above a certain threshold point ($c_{i,j}$).

Idea:

Perform model selection with respect:

1. The exogenous variables Z_i (addition, deletion, replacement moves)
2. The threshold points $c_{i,j}$ of each variable (split and merge moves)

Other specifications:

- threshold points $c_{i,j}$ are defined according to the observed distinct values in the sample of the ith exogenous variable. That is, we expect to have a different regime if the announcement is close to as expected (i.e. Z_i takes small values) or rather unexpected (Z_i takes large values).
- $\gamma_{i,j} \geq -1$ and $s_i \geq -1$ so as not to allow a jump to cause negative variance.
- $q = 1$, so as $s_i S_{i,q}$ to account for the effect the day before the announcement.
- we estimate only a constant for the mean equation, $X = I$.

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Bayesian inference

For a given model m, including κ out of possible K exogenous variables Z_i, $i = 1, ..., K$, the parameter vector includes the coefficients β, σ^2 and $\theta_m = \left\{ \{g_{i,j}, \rho_{i,j}, c_{i,j}\}_{j=1}^{J_i}, \varsigma_i \right\}_{i=1}^{\kappa}$, where:

$$
\begin{aligned}
g_{i,j} &= \log(\gamma_{i,j} + 1) \\
\varsigma_{i,j} &= \log(s_{i,j} + 1) \\
\rho_{i,j} &= \log(r_{i,j})
\end{aligned}
$$

are the transformed coefficients so that $\gamma_{i,j} \geq -1$, $s_i \geq -1$, $r_{i,j} \geq 0$. Note that for every Z_i included in the model we have a collection of threshold points $c_{i,j}$ which is also estimated.

Priors:

- $\{\beta_m, \sigma^2\}$: Normal Inverse Gamma distribution $NIG(a, d, M, V)$, with $M = 0$, $V = I$ and $\alpha = 10e - 6$ and $d = 10e - 6$.
- $\{g_{i,j}, \varsigma_i\}$: N(0,0.4) (allow a jump to more than triple the daily variance).
- $r_{i,j}$: N(0,2) (a jump may last from 1 day up to 1 month).
- $c_{i,j}$: discrete uniform on the distinct values of Z_i observed in the dataset.
- m: discrete uniform.

Population-based MCMC

Evolutionary Monte Carlo (Liang & Wong, 2001) - Parallel tempering (Geyer, 1991)- Adaptive directional sampling (Gilks *et al.*, 1994) - Conjugate gradient Monte Carlo (Liu *et al.*, 2000). For the transdimensional case, Jasra. A.; Stephens, D.A.; Holmes, C.C. (*Biometrika*, 2007).

Designed for problems with complex multi-modal distributions, where standard (vanilla) samplers may fail to move around the support of the target. Consider a sequence of parallel chains $i = 1, ..., N$, that intercommunicate in various ways, with densities $\pi_i(x_i)$, where x_i denotes the current state of the chain i. For the auxiliary distributions π_i we take $\pi_i \propto \pi^{\zeta_i}$,where ζ_i, $1 = \zeta_1 > ... > \zeta_N > 0$, are the inverse temperature parameters. We may take i.e. $\zeta_i = z^{i-1}$.

Moves of the algorithm:

- Mutation. Select a chain i with probability τ_i and perform one sweep of the RJ algorithm.
- Every T iterations make a random choice between performing an exchange move or a crossover move.

Population-based MCMC continued...

- The *exchange move* is the typical move used to exchange information between two parallel tempered chains. At iteration τ we select two adjacent chains (in terms of the temperature parameter ζ_i) uniformly at random and propose to swap their values. To merit reasonable interaction, the temperature ladder is set so that this move is accepted about half of the time (Liu, 2001).The acceptance ratio for the exchange move (analogue for the crossover move):

$$\alpha = min\left(1, \frac{\pi_i(x_j)\pi_j(x_i)}{\pi_i(x_i)\pi_j(x_j)}\right)$$

- In the *crossover move* we transfer a randomly chosen number of variables, along with their associated parameters, to the other chain. The chains need not to be adjacent.

Proposed algorithm

Initialise the parallel chains and sweep over the following:

- Every 10 iterations make a random choice, with equal probability (0.5) between performing an exchange move or a crossover move.
- Otherwise perform a mutation move. For all the parallel chains sweep over the following:
  1. Randomly perform an addition, deletion or replacement move.
  2. Randomly perform a split or merge move for every i.
  3. Update all ς_i as a block for all i's.
  4. For every i update all $g_{i,j}$ as a block for all j's.
  5. For every i update all $\rho_{i,j}$ as a block for all j's.
  6. Draw β and σ^2 from full conditionals.

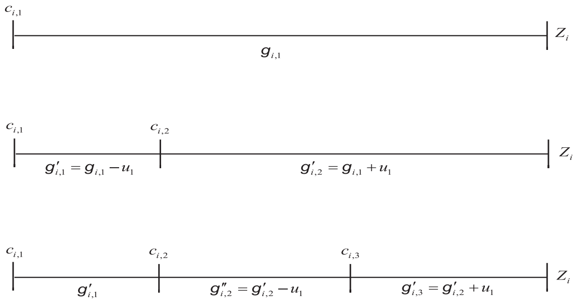
End

- Step 1 involves standard RJ moves (as in Dellaportas & Forster (1999)). The proposal to add a new variable is taken uniform.
- Steps 3,4,5 are standard random-walk M-H kernels.

Split move

Steps:

1. Propose with uniform probability to add a threshold point not present:
$$q(c_i' \mid c_i) = 1/(J_{i,\max} - J_i),$$
where J_i is the number of thresholds present in the current state and $J_{i,\max}$ the total number of distinct values of the ith variable.
2. Draw a random number u_1 and set $g_{i,j}' = g_{i,j-1} + u_1$ and $g_{i,j-1}' = g_{i,j-1} - u_1$. Draw also a random number u_2 and set $\rho_{i,j}' = \rho_{i,j-1} + u_2$ and $\rho_{i,j-1}' = \rho_{i,j-1} - u_2$.



Split and merge move details...

- The proposals for u_1 and u_2 are taken equal to the priors of g and ρ respectively.
- The Jacobian term included in the M-H acceptance ratio for the coefficients $\gamma_{i,j}$ is given as:

$$|J| = \left| \frac{\partial(\gamma')}{\partial(\gamma, u_1)} \right| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2.$$

  The same Jacobian stands for the coefficients $r_{i,j}$. Thus in the M-H acceptance ratio we include the term $|J|^2 = 4$.
- The merge move is set as the inverse of the split move. First we select a threshold point to be deleted with uniform probability. Then without drawing any new coefficients we set the new coefficients as the average of the two old ones (which is the inverse function of that applied to the spit move). That is, $g_{i,j-1} = \frac{g_{i,j}' + g_{i,j-1}'}{2}$ and $\rho_{i,j-1} = \frac{\rho_{i,j}' + \rho_{i,j-1}'}{2}$. Then the inverse Jacobian compared to that of the split move is equal to:

$$\left| J^{-1} \right| = \begin{vmatrix} 0.5 & 0.5 \\ -0.5 & 0.5 \end{vmatrix} = 0.5.$$

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- Use the idea in the Spline-GARCH model of Engle and Rangel (2008), where the variance is modeled by two components, a GARCH(1,1) process G_t, that is related to past returns and a nonlinear (spline) specification E_t, that is related to time and other exogenous variables.
- Simply replace the spline term with our threshold specification.

The Threshold-GARCH Model

$$y_t = X_t\beta + \epsilon_t, \quad \epsilon_t \sim N(0, G_tE_t)$$

$$G_t = (1 - \alpha_1 - \alpha_2) + \alpha_1 \frac{\epsilon_{t-1}^2}{E_{t-1}} + \alpha_2 G_{t-1}$$

$$E_t = \sigma^2 \left[1 + \sum_{i=1}^{K} \sum_{j=1}^{J_i} ([\gamma_{i,j} \exp(-r_{i,j}(t - \lambda_{i,t}))] I_{i,j}) + \sum_{i=1}^{K} (s_i S_{i,q}) \right],$$

where G is a $n \times 1$ vector, with $G_1 = 1$, representing the effect of the GARCH process on the variance structure and α_1, α_2 representing the ARCH and GARCH coefficients respectively.

Explain: $G_t = (1 - \alpha_1 - \alpha_2) + \alpha_1 \frac{\epsilon_{t-1}^2}{E_{t-1}} + \alpha_2 G_{t-1}$.

By using the normalisation of the constant term $(1 - \alpha_1 - \alpha_2)$, the unconditional volatility depends exclusively on the threshold function, since $E(G_t) = 1$. Furthermore in this way we avoid identifiability problems on estimating a constant for the GARCH equation along with the scalar σ^2.

Other details:

- The only difference in the algorithm: Additional M-H kernels to update β, σ^2 and α_1, α_2 since marginalization out of the posterior is not possible.
- The priors for the coefficients $\{\beta, \sigma^2, \theta_m\}$ and the model space m are identical to the Threshold model. For the GARCH coefficients α_1, α_2 we use an uninformative $U(0,1)$ prior. The proposal for the random-walk M-H kernel is truncated normal, such that $0 \leq \alpha_1 < 1$ and $0 \leq \alpha_2 < 1 - \alpha_1$.

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☞ To validate the relative performance of our models we must compare them with a nonlinear model for volatility that uses the same data (macroeconomic news). For this purpose we propose a modification of the Spline-GARCH model of Engle and Rangel (2008). The main difference stands in that the exponential spline is not applied to equally spaced time intervals, but to the instances announcements are made. Furthermore we use a PopulationRJ algorithm to estimate the number of knots included in this spline.

The proposed Spline-GARCH model

$$y_t = X_t\beta + \epsilon_t, \quad \epsilon_t \sim N(0, G_t E_t)$$

$$G_t = (1 - \alpha_1 - \alpha_2) + \alpha_1 \frac{\epsilon_{t-1}^2}{E_{t-1}} + \alpha_2 G_{t-1}$$

$$E_t = \sigma^2 \exp\left(w_0 t + \sum_{i=1}^{k} \left[w_i \left((t - \lambda_{i,t})_+ \right)^2 \right] + \sum_{i=1}^{k} \left[\gamma_i Z_{i,\lambda_{i,t}} \right] + \sum_{i=1}^{k} [s_i S_{i,q}] \right),$$

where $\lambda_{i,t}$ is the time of the last observation of Z_i before time t, $S_{i,q}$ is an indicator variable taking value 1 in the time interval $[t-q, ..., t-1]$ when Z_i is observed at time t and the spline term is specified as:

$$(t - \lambda_{i,t})_+ = \begin{cases} (t - \lambda_{i,t}) & \text{if } t > \lambda_{i,t} \\ 0 & \text{otherwise} \end{cases}$$

In this way when a variable Z_i is included in the model a new knot is generated, causing shifts in the variance process.

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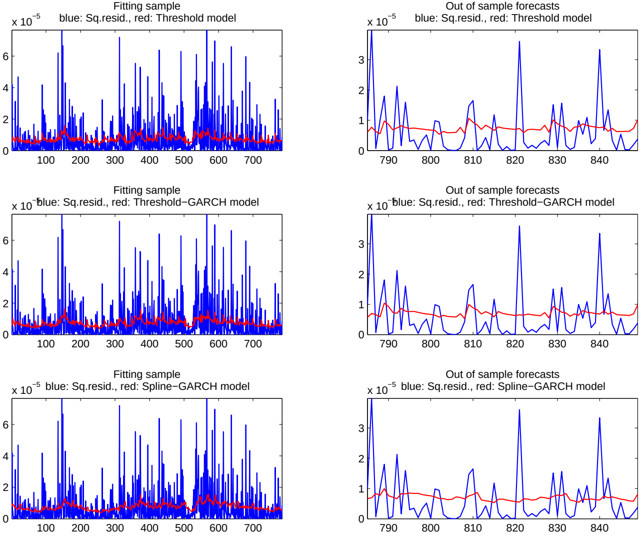
Design of the application

Compare models:

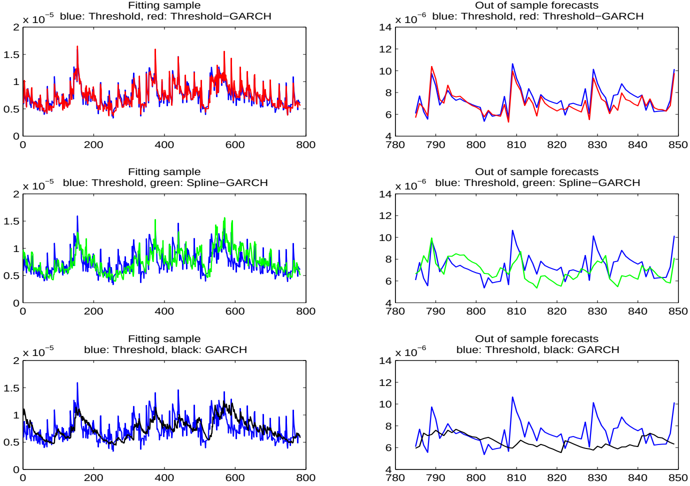
1. Threshold
2. Threshold-GARCH
3. Spline-GARCH
4. a standard GARCH(1,1)

- The models are compared based on various diagnostics both in the sample and out the sample. For the out of sample observations the forecasts are evaluated on basis of the squared residuals as well as the realized volatility. The realized volatility is calculated (as in Andersen et al. (2001)) as the cumulative intraday squared log-returns of each day. The frequency of the intraday data is 5 minutes.
- The PopulationRJ algorithm in all cases runs so as to produce 1.5 million sample, while additional 500,000 iterations are used for burn-in period. Taking in consideration the efficiency-cost trade off we use 5 parallel chains, 4 of them tempered so as to have enough chains exploring the state space at higher temperatures, with a logical computational cost. The temperature ladder, $\zeta_{chain} = z^{chain-1}$, $chain = 1, \ldots, 5$ is calibrated adaptively during the burn-in period of the algorithm.
- The out of sample forecasts are calculated by performing model averaging, over the 10 best models visited and all the threshold point combinations of each model.

Variance estimates vs. squared residuals

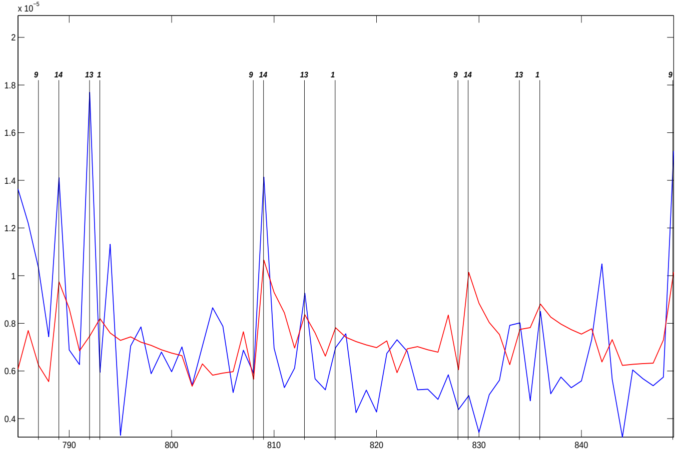


Comparing variance estimates



List of Best Models						
Model	Threshold		Threshold-GARCH		Spline-GARCH	
	Variables	Est. Prob.	Variables	Est. Prob.	Variables	Est. Prob.
First best	1,9,13,14	0.00578	1,9,14	0.00877	7,8	0.01741
Second best	1,4,9,13,14	0.00544	3,9,13,14	0.00691	3,8	0.01328
Third best	9,13,14	0.00542	9,13,14	0.00639	4,14,15	0.01217
Fourth best	3,9,13,14	0.00503	13,14	0.00636	14,15	0.01162
List of macroeconomic announcements included in the four best models of all specifications:						
1	Advance Retail Sales					
3	Consumer Price Index					
4	Durable Goods Orders					
7	Industrial Production					
8	ISM Manufacturing					
9	ISM Non-Manufacturing					
13	Trade Balance					
14	Unemployment rate					
15	Wholesale Inventories					

Threshold model variance forecast vs. realized volatility



blue: Realized volatility, red: Variance forecast, - Announcements (First Best Model)

Forecast errors $\left(\sum_{t=1}^{n}(\epsilon_t^2 - \sigma_t^2)^2\right)$			
Threshold	Threshold-GARCH	Spline-GARCH	GARCH
Fitting sample; compared to squared residuals			
9.7804E-08	1.0025E-07	1.0712E-07	1.0933E-07
Out of sample forecasts; compared to squared residuals			
4.5617E-09	4.7352E-09	4.8572E-09	4.9342E-09
Out of sample forecasts; compared to realized volatility			
4.9190E-10	**4.8440E-10**	5.4750E-10	5.5140E-10

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The idea of Dellaportas & Pourahmadi (2004):

- Consider there are L dependent variables y_l, each of size $n \times 1$, and the stacked vector $Y = [y_1, y_2, ..y_L]'$, where $Y_t \sim N(0, \Sigma_t)$.
- Consider regressing each y_l on its predecessors $y_1, ..., y_{l-1}$:
  $$y_l = \sum_{k=1}^{l-1} (\phi_{lk} y_k) + \epsilon_l,$$
  where ϕ_{lk} are the regression coefficients and σ_l^2 the time-varying variance of each ϵ_l. By convention, $y_1 = \epsilon_1$.
- The above equation in matrix form is written as:
  $TY = \epsilon$,
  where T is a unit lower triangular matrix with $-\phi_{lk}$ in the (l, k)th position.
- Denote with $V = diag(\sigma_1^2, ..., \sigma_L^2)$. Then it follows that T diagonalizes Σ:
  $T\Sigma T' = V$, therefor
  $\Sigma = T^{-1} V (T^{-1})'$.
- This is a Cholesky decomposition, where positive definitiveness is guaranteed.
  Thus we only have to estimate the L univariate σ_l^2 and the coefficients ϕ_{lk} !!!
  Trivial: The likelihood of Y is normal with variance-covariance matrix $T^{-1} V (T^{-1})'$.
  Simply apply our Threshold-GARCH model for each univariate σ_l^2.

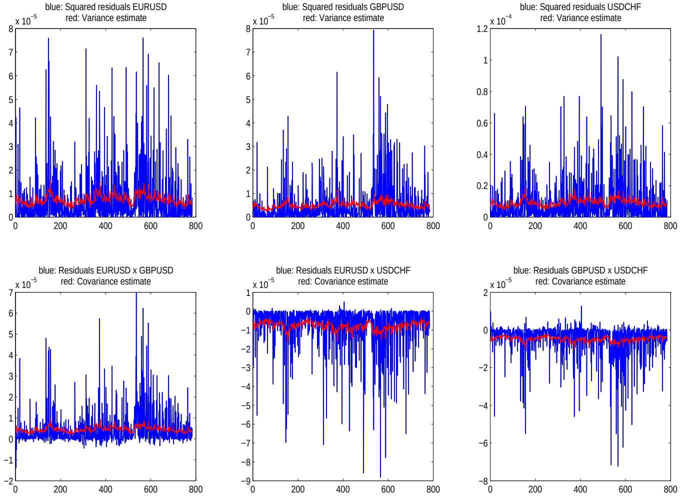
The algorithm:

Initialise the chain and sweep over the following:

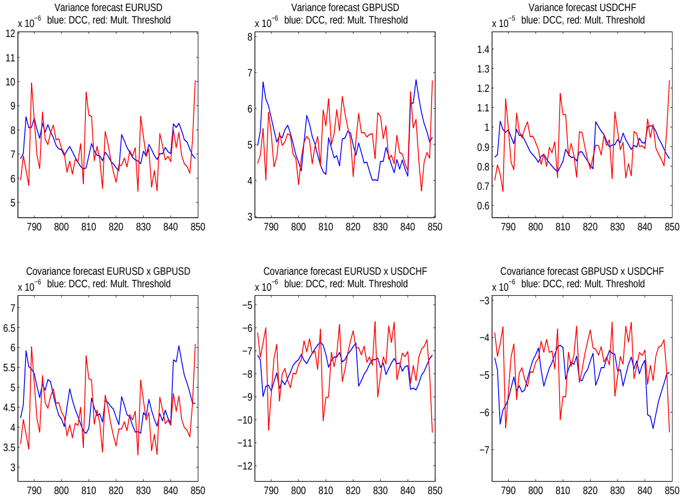
- Every 10 iterations make a random choice, with equal probability (0.5) between performing an exchange move or a crossover move.
- Otherwise: For *chain* $= 1 : 5$
  - For $l = 1 : L$
    1. Randomly perform an addition, deletion or replacement move.
    2. Randomly perform a split or merge move for every i.
    3. Update σ^2
    4. Update α_1, α_2
    5. Update all ς_i as a block for all i's.
    6. For every i update all $g_{i,j}$ as a block for all j's.
    7. For every i update all $\rho_{i,j}$ as a block for all j's.
  - End L
  - Update ϕ
- End *chains*

We use $L = 3$ exchange rates, EURUSD, GBPUSD and USDCHF for the same time interval. Our model is compared to the DCC model of Engle (2002) both in the sample and out the sample using the 5-minute intraday realized variances and covariances.

Variance-covariance estimates vs. squared residuals



Comparing variance forecasts



Forecast errors		
	Multivariate Threshold	DCC
	Fitting sample; compared to squared residuals	
Var. EURUSD	**1.0092E-07**	1.0951E-07
Var. GBPUSD	**5.3764E-08**	5.4988E-08
Var. USDCHF	**1.5510E-07**	1.6529E-07
Cov. EURUSD,GBPUSD	**5.4355E-08**	5.6266E-08
Cov. EURUSD,USDCHF	**1.1480E-07**	1.2398E-07
Cov. GBPUSD,USDCHF	**6.6109E-08**	6.8186E-08
	Out of sample forecasts; compared to squared residuals	
Var. EURUSD	**4.8214E-09**	4.9424E-09
Var. GBPUSD	**3.0391E-09**	3.1334E-09
Var. USDCHF	**1.0522E-08**	1.0544E-08
Cov. EURUSD,GBPUSD	**3.4468E-09**	3.5204E-09
Cov. EURUSD,USDCHF	**6.7549E-09**	6.8854E-09
Cov. GBPUSD,USDCHF	**4.4272E-09**	4.5062E-09
	Out of sample forecasts; compared to realized volatility	
Var. EURUSD	**4.9190E-10**	5.4070E-10
Var. GBPUSD	2.8780E-10	**2.7320E-10**
Var. USDCHF	**8.4990E-10**	8.9040E-10
Cov. EURUSD,GBPUSD	**2.7560E-10**	3.0550E-10
Cov. EURUSD,USDCHF	**7.5840E-10**	8.5410E-10
Cov. GBPUSD,USDCHF	**3.0050E-10**	3.5730E-10

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